Alfvén wave dissipation via electron energization

Andrew N. Wright,[1] W. Allan,[2] and Peter A. Damiano[1]

[1] It has recently been noted that FAST (Fast Auroral SnapshoT) mission data of auroral current systems associated with Alfvén waves has an electron kinetic energy flux density that is similar to the Poynting vector [*Chaston et al.*, 2002]. In Ultra-Low-Frequency (ULF) wave theory, which considers global wave modes of the magnetosphere (with frequency 1–5 mHz), the traditional dissipation mechanism is taken to be Joule heating associated with ionospheric currents that are fed by the Poynting vector. Hence FAST observations indicate that electron acceleration can supply an additional sink of energy that is of similar magnitude to the traditional one. In this letter we use typical Pc5 parameters to estimate the importance of electron acceleration as a sink of Alfvén wave energy for these global waves. We find that the electron dynamics must be treated nonlinearly, and that electron acceleration drains a similar amount of energy from the Alfvén wave fields as ionospheric dissipation, and for some events may actually exceed the latter. *INDEX TERMS:* 2752 Magnetospheric Physics: MHD waves and instabilities; 2716 Magnetospheric Physics: Energetic particles, precipitating; 2451 Ionosphere: Particle acceleration. **Citation:** Wright, A. N., W. Allan, and P. A. Damiano, Alfvén wave dissipation via electron energization, *Geophys. Res. Lett.*, *30*(16), 1847, doi:10.1029/2003GL017605, 2003.

1. Introduction

[2] Alfvén waves standing on closed magnetic field lines are a common feature of the terrestrial magnetosphere. Recent observations linking optical auroral emissions from the ionospheric boundary of such waves [*Xu et al.*, 1993; *Samson et al.*, 1996; *Wright et al.*, 1999 and references therein] have led to an interest in electron acceleration from the Ultra-Low-Frequency (ULF) wave community. For example, the field aligned current density above the ionosphere is a few $\mu A\ m^{-2}$, and requires the electrons to have an energy of the order of several keV. It is the collision of these energetic electrons with the dense ionosphere that produces the optical auroral emission, and the acceleration process is likely to share common physics with the much studied field of electron acceleration in global magnetospheric current systems and aurorae. Indeed, recent papers by *Rankin et al.* [1999], *Rönnmark* [1999, 2002], *Rönnmark and Hamrin* [2000], *Wright et al.* [2002], and *Wright and Hood* [2003] have stressed this link, and those papers by

Rönnmark and Wright have noted the importance of nonlinear electron dynamics.

[3] The traditional approach to modeling ULF Alfvén waves has been the single fluid magnetohydrodynamic (MHD) limit, and this has proved to be very successful. Within this description dissipation is provided by a finite ionospheric Pedersen conductivity which causes freely oscillating Alfvén waves to decay in time, and limits the width of resonantly driven Alfvén waves (commonly referred to as Field Line Resonances, FLRs). Recent FAST observations by *Chaston et al.* [2002] have indicated that the energy flux density carried by the electrons is similar to the Poynting vector of the Alfvén wave fields. If this is true of ULF Alfvén waves, the energization of electrons required to carry the field aligned current would represent a significant loss of Alfvén wave energy and lead to wave decay even if the ionosphere is perfectly conducting. Indeed, a recent study by *Vaivads et al.* [2003] shows that the Poynting flux observed in the magnetosphere by Cluster, when mapped to the ionosphere, is similar to the electron energy flux recorded by DMSP on a common field line mapping to the outer plasma sheet. They interpret this as a global Alfvén wave current system similar to a Pc5 ULF wave.

[4] This letter assesses the importance of electron energization for the energy budget of global ULF Alfvén waves, which are quite different from the higher frequency and smaller scale "ionospheric Alfvén resonator" modes that exist only at low altitudes. We begin by reviewing energetics in the single fluid limit (in which the electron mass is neglected) and then describe how the situation is modified by finite electron mass through using the two-fluid approximation.

2. Governing Equations

[5] We adopt an axisymmetric nonuniform magnetic field (**B**) that is at rest as our equilibrium state. The Alfvén wave magnetic field (**b**) only has an azimuthal component and its electric field (**E**) lies in a meridional plane. Combining the linear ion momentum equation, the nonlinear electron momentum equation, continuity and induction equations gives the energy continuity equation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}nm_e v_e^2 + \frac{1}{2}nm_i v_i^2 + \frac{b^2}{2\mu_o}\right) + \boldsymbol{\nabla}\cdot\left(\frac{1}{2}nm_e v_e^2 \mathbf{v}_e + \frac{\mathbf{E}\times\mathbf{b}}{\mu_o}\right) = 0 \quad (1)$$

($\mathbf{v}_e$ = electron fluid velocity; $\mathbf{v}_i$ = ion fluid velocity; m_e = electron mass; m_i = ion mass; n = ion or electron number density, which are the same as the plasma is quasi-neutral). Note that we have neglected the thermal energy of the electron fluid in this derivation. Retaining the nonlinear

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[1]Mathematical Institute, University of St. Andrews, St. Andrews, Fife, UK.
[2]National Institute of Water and Atmospheric Research, Wellington, New Zealand.

$(\mathbf{v}_e \cdot \nabla)\mathbf{v}_e$ term in the electron momentum equation leads to the kinetic energy density flux term in (1).

3. Single Fluid Limit

[6] In the single fluid MHD approximation we let $m_e/m_i \rightarrow 0$ (see *Wright and Allan* [1996] and references therein for a detailed account) and may identify the single fluid velocity (**V**) with $\mathbf{v}_i$ and the mass density (ρ) with nm_i. Thus (1) becomes

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho V^2 + \frac{b^2}{2\mu_o}\right) + \nabla \cdot (\mathbf{E} \times \mathbf{b}/\mu_o) = 0 \qquad (2)$$

[7] For perfectly conducting ionospheres **E** and **V** have nodes at the ionospheric ends of the field lines. Figure 1 indicates how the wave energy of the fundamental mode is exchanged between kinetic energy (concentrated around the central section) and magnetic energy (concentrated towards the ends) throughout the cycle. The transport of energy between these two regions is described by the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{b}/\mu_o$ and the single fluid ideal Ohm's law $(\mathbf{E}\} = -\mathbf{V} \times \mathbf{B})$ means **S** is strictly field-aligned. The values of E and b just above the ionosphere are related by

$$b^{\pm} = \pm\mu_o\Sigma_p E_{\perp}^{\pm} \qquad (3)$$

(The $\pm$ sign denoting the northern/southern end.)

[8] In the limit of infinite Σ_p we find $E_{\perp}^{\pm} \rightarrow 0$ so the Poynting vector vanishes at the ionospheric boundary and there is no loss of wave energy from the flux tube. Each L-shell oscillates at its own frequency ω_A (L). For finite Σ_p the field aligned Poynting vector ($\pm (b^{\pm})^2/\mu_0\Sigma_p$) is always directed into the ionosphere, where Joule heating occurs, and causes the Alfvén wave to damp over typically 2–10 cycles [*Newton et al.*, 1978; *Allan and Knox*, 1979; *Allan and Wright*, 1997].

[9] The electrons, which constitute a massless charge neutralizing fluid in this approximation, are still accelerated to carry the field aligned current but have vanishing kinetic energy. Figure 2 is a schematic of the upward current region. The section above the B/n peak is the magnetosphere and the converging field geometry here means $j_{\parallel}$ increases proportionally with B, as n does not change substantially here. This leads to an increase in $v_{e\parallel} = -j_{\parallel}/(ne)$ as indicated
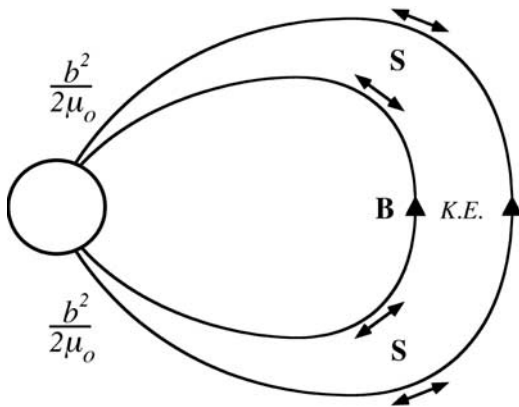

Figure 1. A dipolar-like closed magnetic flux tube supporting an Alfvén wave has a Poynting vector (**S**) that is strictly field aligned in the single fluid MHD approximation.
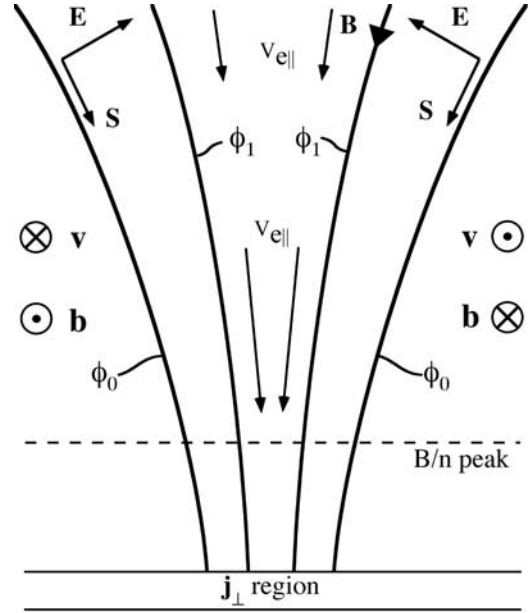

Figure 2. A schematic of the converging magnetic field geometry that exists above the ionosphere. Below the B/n peak the F region density increases. The field aligned current is carried mainly by electrons whose speed $v_{e\parallel}$ is required to increase as they approach the B/n peak. In the single fluid MHD limit **S** remains field aligned, and for a quasi-steady wave ϕ is constant along **B**.

by the arrow size in Figure 2. The electron transit time across the acceleration region (which extends for about 1 R_E above the B/n peak - see *Wright and Hood* [2003]) is less than the Alfvén wave period by two orders of magnitude, so we represent the Alfvén wave fields as steady during the upward current phase shown in Figure 2. The normal direction to the plane of the figure is the azimuthal direction. Since **E** is perpendicular to **B**, magnetic field lines are also contours of electrostatic potential, ϕ. The field aligned currents ultimately reach the E region where they are diverted into perpendicular currents. Between the E region and the B/n peak is the F region, which is dominated by abundant collisionless ionospheric plasma.

4. Two-fluid Approximation

[10] To study electron dynamics in more detail we use the two-fluid approximation. The scaling of the mean field-aligned electron drift velocity as B/n was first recognised by *Swift* [1975], and peaks at an altitude of typically 0.5–1 R_E [*Lysak and Hudson*, 1979]. The large value of $v_{e\parallel}$ here results in the inertial term in the electron momentum equation becoming important [*Goertz and Boswell*, 1979; *Rönnmark*, 1999, 2002; *Wright et al.*, 2002], and now leads us to consider whether the electron kinetic energy density (or its flux) may be important. To estimate the effect of electron energization on energy balance in the Alfvén wave, we now retain finite m_e and use equation (1). The ratio of electron kinetic energy density to, for example, magnetic field energy density is

$$\frac{\frac{1}{2}nm_e v_{e\parallel}^2}{b^2/2\mu_o} \approx \frac{\lambda_e^2}{\lambda^2} \qquad (4)$$
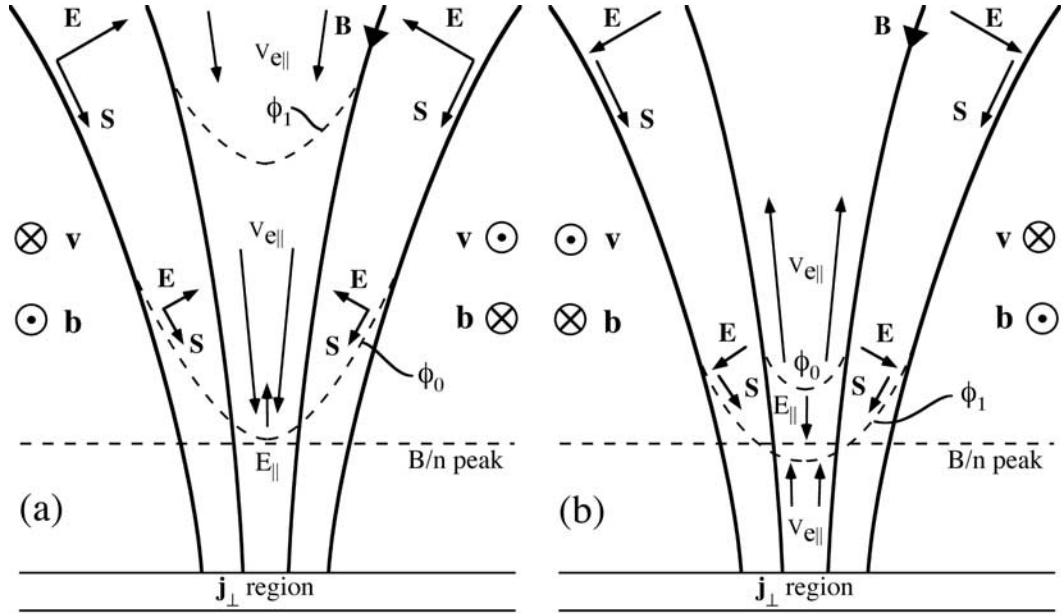
Figure 3. (a) The same as the upward current case in Figure 2, but including finite m_e meaning $E_\parallel$ is required to accelerate the electrons. The potential contours are U-shaped and the Poynting vector (aligned with these contours) feeds energy into the acceleration region. (b) similar to (a) but for a downward current.

where λ_e is the electron inertial length (skin depth) and is equal to c/ω_{pe}. ($\omega_{pe}^2 = ne^2/m_e\varepsilon_o$.) λ is the latitudinal wavelength (i.e., the perpendicular wavelength in the North-South direction), so the ratio in (4) will have its maximum near the B/n peak. In evaluating (4) we used $j_\parallel \simeq -nev_{e\parallel}$ and $\mu_0 j_\parallel \simeq (\nabla \times \mathbf{b}) \cdot \mathbf{B}/B$. Assuming a constant n of 1 cm^{-3} in the magnetosphere gives $\lambda_e \simeq 5$ km, while the typical latitudinal wavelength of an $L = 10$ ULF Alfvén wave is about 25 km [*Wright et al.*, 2002]. Thus the ratio in (4) is roughly 0.05 suggesting that only a small fraction of the total energy resides in the electrons.

[11] Following *Chaston et al.* [2002] we now study the relative importance of electron energy flux density compared with the Poynting vector at the B/n peak.

$$\frac{\frac{1}{2}nm_e v_{e\parallel}^2 v_{e\parallel}}{E_\perp b/\mu_o} \approx \frac{1}{2}\mu_o\Sigma_p v_{e\parallel}\left(\frac{\lambda_e}{\lambda}\right)^2 \qquad (5)$$

In calculating (5) we have used the relation in (3) and estimated the Poynting vector based upon the single fluid MHD model. The ratio $(\lambda_e/\lambda)^2$ again appears in (5) and from (4) we expect this to be small. However, there exists the possibility that Σ_p and $v_{e\parallel}$ can be large. Indeed, a typical Σ_p is 5 S and electrons of keV energies are associated with $j_\parallel$ of a few μA m^{-2}, both of which are consistent with $v_{e\parallel} \simeq 2 \times 10^7$ m s^{-1}. For $b^\pm = 100$ nT and $\Sigma_p = 5$ S equation (3) gives $E_\perp^\pm = 16$ mV/m which is consistent with ion drift velocities of 320 m s^{-1} in the ionosphere. These figures suggest the ratio in (5) is about 3, so the energy transported by electrons is certainly comparable to the Poynting flux, and will represent a significant drain on Alfvén wave energy which we can investigate by using the parallel component of the generalised Ohm's law. *Wright et al.* [2002] show this to be of the form

$$E_\parallel \approx -\frac{m_e}{e}(v_{e\parallel}\nabla_\parallel)v_{e\parallel} \qquad (6)$$

in the acceleration region: The neglect of $\partial v_{e\parallel}/\partial t$ relative to $(v_{e\parallel}\nabla_\parallel)v_{e\parallel}$ is actually the opposite of what is traditionally assumed [e.g., *Goertz and Boswell*, 1979]. The dominance of the $(v_{e\parallel}\nabla_\parallel)v_{e\parallel}$ term for low frequency waves arises through the scale length of our nonuniform (dipolar) magnetic field, even if $v_{e\parallel}$ is much less than the Alfvén speed [*Wright et al.*, 2002]. From (6) we see that $\mathbf{E}$ now has a parallel component which accelerates the electrons. Since

$$\mathbf{E} = -\nabla\phi \qquad (7)$$

magnetic field lines are no longer potential contours, and the situation depicted in Figure 2 (for $m_e/m_i \to 0$) needs to be revised as shown in Figure 3a for finite m_e. The form of $j_\parallel$ (and hence $v_{e\parallel}$) is the same as in Figure 2, but we need to do work to energise the electrons when m_e is finite (Figure 3a). The electrons enter at the top of the figure and begin to cross the modified potential contours as they move downward, thus being accelerated. $\mathbf{E}$ is perpendicular to ϕ contours, and at high altitudes (where $v_{e\parallel}$ is small) is simply associated with $\mathbf{V} \times \mathbf{B}$ as in the single fluid model. In the middle of the accelerator region ϕ contours are U-shaped and $\mathbf{E}$ is parallel to $\mathbf{B}$. The form of these contours is as depicted by *Mozer et al.* [1977]. In Figure 3 we map the contours onto an ideal Alfvén wave solution in the magnetosphere, thus giving some understanding of the global form of these contours above the acceleration region. The exchange of energy from the Alfvén wave fields to the electrons is clearly seen with the Poynting vector,

$$\mathbf{S} = \mathbf{E} \times \mathbf{b}/\mu_o = -(\nabla\phi) \times \mathbf{b}/\mu_o \qquad (8)$$

Thus $\mathbf{S}$ lies along ϕ contours, and so the form of ϕ in Figure 3a indicates that electromagnetic energy is fed into the acceleration region from either side of the current layer and then exits from the bottom of the acceleration region as a field aligned electron energy flux. The situation in Figure 3a

is for an upward current (downgoing electrons), and two-fluid and kinetic studies indicate the acceleration region extends for about 1 R_E above the B/n peak [*Rönnmark*, 1999, 2002; *Wright et al.*, 2002; *Wright and Hood*, 2003]. The situation for a downward current will be quite different. *Rönnmark* [1999] notes that for the same magnitude of $j_\parallel$, the energy of electrons near the B/n peak will be similar for currents flowing in either direction since $v_{e\parallel} = -j_{e\parallel}/ne$. The main difference is that the downward current is carried by ionospheric electrons being accelerated upward, and *Temerin and Carlson* [1998] suggest this will occur near the B/n peak but over a much smaller scale (probably related to the ionospheric density scale height, see *Wright et al.* [2002]) than the upward current scale of $1R_E$. The downward current situation is depicted in Figure 3b. The electron energy flux should be similar to that in Figure 3a, so will represent a similar drain on the Alfvén wave energy.

5. Discussion and Conclusions

[12] As the Alfvén wave cycle proceeds, we switch between the upward and downward current phases over a timescale of hundreds of seconds. The picture that emerges is one of the global Alfvén wave oscillation squirting magnetospheric electrons at high energies into the ionosphere for half a cycle. During the next half cycle the electrons that have just been lost from the magnetosphere are replaced by the Alfvén wave sucking up cold ionospheric electrons into the magnetosphere at similarly high energies. For finite m_e the electron acceleration represents a sink of the Alfvén wave energy comparable with the dissipation associated with the Pederson conductivity.

[13] The fate of the downgoing electrons will be to collide with ionospheric particles where they will lose their energy and may stimulate auroral emissions and feed the perpendicular ionospheric current systems. The fate of the upgoing electrons depends upon the topology of the magnetic field line. If the downward current is on an open magnetic field line, the electrons may stream off indefinitely or may be affected by the field in the generator region. If the field line is closed (as in the scenario in Figure 1) it is likely that counterstreaming beams will be produced. These beams could be unstable and scatter, or be influenced by electric fields in the equational section that act to localise electrons there to balance the ion charge density associated with the divergence of the polarization current [*Wright et al.*, 2002]. Whatever the case, it is evident that upward accelerated electrons do not represent a loss of energy from the magnetospheric flux tube in the way that downgoing electrons do. The upward electron energy may well by thermalised via microscopic instabilities and thus manifest as a heated electron population. Thus it appears that the drain of Alfvén wave energy throughout the cycle goes alternately into heating the ionosphere and then the magnetosphere. These possiblities are currently being investigated in more detail.

[14] A set of closed field lines that is supporting standing Alfvén waves is likely to have a weak phase variation with L (λ is large) to begin with. Such a system will initially have a small $j_\parallel$ suggesting modest $v_{e\parallel}$ is required. Thus it is likely that at early times the ratio in (5) is less than one indicating the traditional ionospheric Ohmic heating is the main dissipation mechanism. At later times the Alfvén waves have phase-mixed, and $\lambda \simeq 2\pi/\omega'_A t$, ω'_A being the derivative of the

Alfvén frequency at ionospheric altitudes [*Wright et al.*, 1999]. The reduction of λ with increasing t means $j_\parallel$ will increase and require $v_{e\parallel}$ to increase accordingly. Hence the ratio in (5) increases with time, and for typical ULF Pc5 pulsations we expect Alfvén wave energy loss through electron acceleration to be comparable with the traditional dissipation mechanism associated with a finite Σ_p. Indeed, electron precipitation in the ionosphere will probably enhance the value of Σ_p leading to a reduced ohmic dissipation and enhanced phase mixing. This will promote smaller λ and larger $j_\parallel$ and $v_{e\parallel}$. It could be that accounting for this nonlinear feedback can give some insight into the formation of narrow current structures that are sometimes observed to be embedded in extended wave fields [e.g., *Elphic et al.*, 1998; *Wright et al.*, 2002].

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W. Allan, National Institute of Water and Atmospheric Research, P.O. Box 14-901, Kilbirnie, Wellington, New Zealand. (w.allan@niwa.cri.nz)

P. A. Damiano and A. N. Wright, Mathematical Institute, University of St. Andrews, St. Andrews, Fife, KY16 9SS, UK. (pdamiano@mcs.st-and.ac.uk; andy@mcs.st-and.ac.uk)