

# Large- $m$ waves generated by small- $m$ field line resonances via the nonlinear Kelvin-Helmholtz instability

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**Abstract.** Recently, ultralow-frequency waves with large azimuthal wavenumber (large  $m$ ) have been observed on similar  $L$  shells and with the same (or similar) frequencies as small- $m$  field line resonances (FLRs). The large- $m$  waves appeared to the west of the small- $m$  FLRs and had westward phase propagation while the small- $m$  FLRs had tailward phase propagation. We propose an extension to an earlier waveguide model to explain these observations. We suggest that small- $m$  tailward propagating waveguide modes drive the small- $m$  FLRs. Phase mixing within these FLRs allows the development of the nonlinear Kelvin-Helmholtz (K-H) instability near the resonant field lines. Phase-mixing scale lengths are limited by ionospheric dissipation, and we show that realistic ionospheric Pedersen conductivities result in the dominance of a single zero-frequency K-H wave in each small- $m$  FLR region having  $m$  consistent with observation of the large- $m$  disturbances. K-H growth rates are significant, but not large enough to disrupt the small- $m$  FLRs. We propose that unstable ion distributions amplify the seed K-H waves as the ions drift westward. This leads to observable large- $m$  drift waves at or beyond the westward limits of the small- $m$  FLR regions.

## Introduction

Ultralow-frequency (ULF) waves in the magnetosphere have been classified in various ways, usually related to the wave frequency or period. In the last decade or two, it has become obvious that the dimensionless azimuthal wavenumber ( $m$ ) is an important indicator of the likely driving mechanism of waves with millihertz frequencies. Small- $m$  waves ( $m$  smaller than about 10) appear to be understandable in purely magnetohydrodynamic (MHD) terms as for example Alfvén field line resonances (FLRs) driven by compressional fast mode waves [e.g., *Southwood*, 1974], which in turn are probably excited by external sources in the solar wind. Such resonant coupling can occur quite happily in a cold plasma. However, the driving mechanism for large- $m$  waves ( $m$  larger than about 15) appears to require a warm plasma with  $\beta \sim 1$  (where  $\beta$  is the ratio of plasma pressure to magnetic pressure). A given wave may be driven by plasma pressure anisotropy, leading to drift mirror and related modes [e.g., *Hasegawa*, 1969; *Pokhotelov et al.*, 1986]; or by a bounce resonance interaction with hot protons [e.g., *Southwood et al.*, 1969; *Chen and Hasegawa*, 1988]; or by several other similar mechanisms. All these mechanisms

require, directly or indirectly, unstable distributions of energetic particles inside the magnetosphere.

Because of the differences in likely driving mechanisms for small- $m$  and large- $m$  waves, it seems unlikely at first sight that there should be any significant correlation between occurrences of the two wave types. However, *Fenrich et al.* [1995] (hereinafter FSSG) observed both types of wave using the Super Dual Auroral Radar Network (SuperDARN) and found that they shared many common properties. Figure 9 of FSSG shows the following interesting features.

1. The small- $m$  modes ( $m < 10$ ) were observed on the dawn and dusk flanks on closed field lines.
2. They had quantized frequencies, and antisunward azimuthal phase velocities of the order of the sheath flow speed.
3. The large- $m$  events ( $10 < m < 50$ ) were seen in the noon-dusk and midnight-dawn quadrants only.
4. They had the same frequencies as and occurred on similar  $L$  shells to the small- $m$  events, but had westward phase velocities.

The small- $m$  waves exhibited typical FLR characteristics. The large- $m$  waves showed similar properties to other reported waves of this type [e.g., *Grant et al.*, 1992], including westward propagation, which is typical of events associated with energetic ion drifts [e.g., *Allan et al.*, 1983], and an FLR-like structure.

FSSG proposed a dispersive magnetospheric waveguide model [e.g., *Walker et al.*, 1992; *Wright*, 1994] to explain the relationship between the two types of wave (a schematic is shown in Figure 13 of FSSG). Sources on the magnetospheric flanks stimulate the waveguide.

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Small- $k_y$  components of the disturbance (where  $k_y$  is the dimensional azimuthal wavenumber in a Cartesian waveguide model) remain close to the source in longitude for a reasonable time, and drive small- $m$  resonances at lower  $L$  values. Large- $k_y$  components propagate in the waveguide in both sunward and antisunward directions. According to FSSG the large  $k_y$  components drive large- $m$  resonances at the same  $L$  values (and therefore the same frequencies) as the small- $m$  resonances in all four quadrants of longitude.

We find this model to be very interesting, but consider that it has the following drawbacks.

1. FSSG noted that according to the waveguide model of Wright [1994], large- $m$  components should have significantly different frequencies to small- $m$  components, and so it is difficult to see why they should appear on similar  $L$  shells with similar frequencies.

2. Effectively only westward traveling large- $m$  waves were observed on the westward sides of the small- $m$  waves, in the midnight-dawn and noon-dusk quadrants. FSSG suggested this may be because of limited data.

In this paper we present an extension of the FSSG waveguide model which we feel overcomes these drawbacks. We invoke the idea of the nonlinear Kelvin-Helmholtz (K-H) instability generated by the velocity shear of the small- $m$  resonances [e.g., Browning and Priest, 1984; Rankin *et al.*, 1993] to generate seeds for amplification by westward drifting energetic ions, thus generating the large- $m$  waves to the west of the small- $m$  waves.

In section 2 we present a schematic of this model summarising the ideas, and in section 3 we give more detailed qualitative and quantitative discussion in support of it.

## Schematic of Model

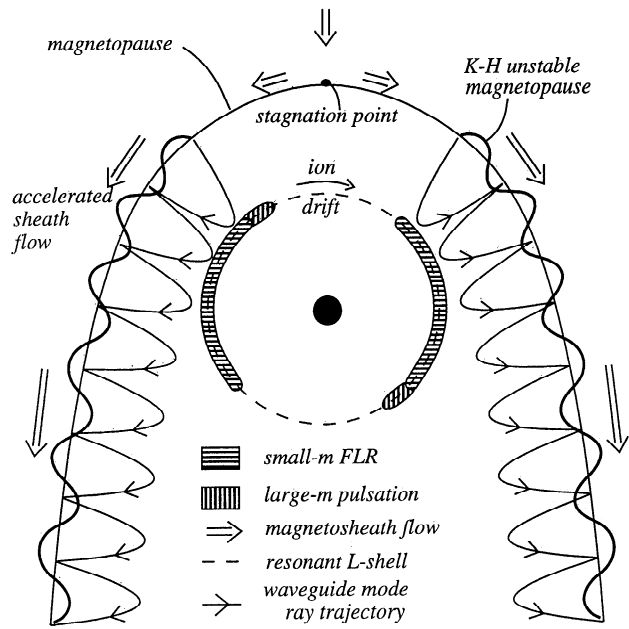
Figure 1 shows a schematic of the process we propose. The steps in the process are as follows.

1. Tailward propagating waveguide modes are stimulated on the flanks of the magnetosphere, either by K-H waves on the magnetopause driven by the accelerated magnetosheath flow, or possibly by pressure pulses in the solar wind.

2. Small- $k_y$  (small- $m$ ) waveguide modes on opposite flanks propagate so slowly tailward that they are able to drive small- $m$  FLRs on earthward field shells whose Alfvén frequency matches the  $m \approx 0$  waveguide mode frequency.

3. Velocity shear across a small- $m$  FLR is large enough to drive a nonlinear K-H instability during a half-cycle of the FLR, and to develop a spectrum of large- $m$  waves. Within a few cycles of the FLR, a balance develops between phase mixing and ionospheric dissipation which chooses a particular fastest-growing  $m$ . The growth rate is small enough not to disrupt the FLR.

4. Unstable distributions of westward drifting ions provide energy to amplify the large- $m$  seed wave in each FLR region. The resulting large- $m$  drift waves emerge from the westward edges of the FLR regions with phase



**Figure 1.** Schematic of the coupling process in the magnetosphere, from K-H wave on the magnetopause, to flank waveguide mode, through small- $m$  FLR (which is unstable to a large- $m$  K-H wave), to large- $m$  ion drift wave.

velocities comparable with the appropriate drift velocities of the resonant ions, and with frequencies chosen by the development of a standing wave structure along the magnetic field. The large- $m$  waves may be observable at this point or may continue to grow to observable size if the associated ion distributions are still unstable.

Note that K-H or pressure pulse disturbances on the magnetopause are both suitable for exciting magnetospheric waveguide modes [Wright and Rickard, 1995]. FSSG observed small- $m$  events exclusively on the flanks, and the K-H mechanism is the best explanation for their data set; the stagnation point around the subsolar point (and consequent stability to K-H modes) explains the dearth of these events around noon. When small- $m$  events are seen around noon [e.g., Ziesolleck and McDiarmid, 1994], the pressure pulse mechanism is the more likely driver.

The processes depicted in Figure 1 allow the existence of small- $m$  and large- $m$  waves on the same  $L$  shell with similar (but not necessarily identical) frequencies. From Figure 1 of Orr and Matthew [1971] the fundamental guided poloidal mode ( $m \rightarrow \infty$ ) eigenfrequency is about 25% smaller than the fundamental axisymmetric toroidal mode ( $m = 0$ ) eigenfrequency at  $L \sim 7$ . Since the large- $m$  and small- $m$  waves are not pure  $m \rightarrow \infty$  and  $m = 0$  modes, their eigenfrequencies are likely to be closer together than those of the pure modes. For a less than optimum observation, a large- $m$  FLR could appear to have the same frequency as a small- $m$  FLR within the data resolution, which may explain this feature in the data of FSSG. For a well-defined observation with optimum data resolution a difference in frequency might be detectable.

Note that the process described above should still occur for FLRs excited around the noon meridian by, for example, pressure-pulse-excited waveguide modes.

## Supporting Discussion

### Small- $m$ Field Line Resonance

We assume that waveguide modes are stimulated on the flanks of the magnetosphere either by magnetosheath-flow-driven K-H waves on the magnetopause, or by solar wind pressure pulses impinging on the magnetopause. As discussed by *Wright* [1994] and *Rickard and Wright* [1994], small- $k_y$  (small- $m$ ) waveguide modes on opposite flanks propagate so slowly tailward that they are able to mode couple with Alfvén waves on Earthward field shells with matching frequency, and therefore can drive small- $m$  FLRs at those  $L$  shells. In order that a driving mechanism for large- $m$  waves can also exist, we must assume  $\beta \sim 1$  at the FLR positions.

### Nonlinear Kelvin-Helmholtz Instability

As time evolves, a magnetospheric FLR develops increasingly finer-scale structure in the radial direction by the process called “phase mixing” [e.g., *Heyvaerts and Priest*, 1983; *Mann et al.*, 1995, Figure 2; *Rickard and Wright*, 1994, Figure 9]. The detuning between the continuously varying eigenfrequencies of the field lines generates a continuously increasing phase shear with time. In the case of an FLR this leads to increasing amplitude and narrowing width around the resonant field line. This process is limited by dissipation effects, dominantly ionospheric dissipation for the magnetosphere [*Mann et al.*, 1995]. The radial phase shear across the resonance region means that a velocity shear develops across the FLR amplitude maximum [e.g., *Hollweg and Yang*, 1988; *Rankin et al.*, 1993]. If this shear is large enough the FLR can become K-H unstable during each half-cycle of the wave. We refer to this as a nonlinear K-H instability as it is a nonlinear modification of the behaviour of a linear FLR.

In their nonlinear numerical simulation, *Rankin et al.* [1993] found that for their chosen parameters the K-H instability disrupted the FLR within a quarter cycle of the oscillation. In the present work we choose parameters based upon observations, and show that the small- $m$  FLRs can survive for many cycles (in agreement with data).

We consider three representative FLRs with frequencies  $f = 1.3, 1.9$ , and  $2.6$  mHz at  $L$  values of 11.0, 9.3, and 8.1, respectively. These are consistent with the stable frequencies and positions observed using SuperDARN as given for example by FSSG. These values can be well represented by an Alfvén frequency variation

$$\omega_A \propto L^{-2.27} \quad (1)$$

allowing the magnitude of the spatial gradient in Alfvén frequency  $\omega'_A$  to be obtained. We also choose maximum  $\mathbf{E} \times \mathbf{B}$  drift velocities  $V_{0M}$  for the FLRs of  $10^5$ ,  $8 \times 10^4$ , and  $6 \times 10^4$  ms $^{-1}$  at  $L = 11.0, 9.3$ , and  $8.1$ , respec-

tively, in the equatorial plane. These values are consistent with maximum drift velocities observed in the ionosphere [e.g., *Walker et al.*, 1992] assuming a fundamental standing Alfvén wave structure and reasonable ionospheric Pedersen conductances. For reference the FLR parameters are given in Table 1.

We make use of the local phase-mixing length  $L_{PH}(t)$  defined by *Mann et al.* [1995] at a position  $x$  (where  $x$  is equivalent to a radial position in the equatorial plane) as

$$L_{PH}(t) = 2\pi[\omega'_A(x)t]^{-1}. \quad (2)$$

The phase mixing length is an excellent estimate of the radial wavelength of the small- $m$  FLRs [see *Rickard and Wright*, 1994, Figure 9] and may be used to estimate the velocity shear within these waves. Figure 2 is a schematic showing the phase-mixing length within a typical FLR radial structure. To determine the maximum growth rate  $\gamma_{max}$  and the azimuthal wavenumber with maximum growth  $k_{max}$ , we consider that the “strong phase mixing” results of *Browning and Priest* [1984] are most applicable to the present case. From section 3 of *Browning and Priest* [1984] we obtain

$$\gamma_{max} = 1.70V_0/L_{PH} \quad (3)$$

$$k_{max} = 3.46/L_{PH} \quad (4)$$

where  $V_0$  is the wave  $\mathbf{E} \times \mathbf{B}$  drift velocity at a given position and time. These expressions give results which are reasonably consistent with the equivalent results determined by *Walker* [1981], when due allowance is made for the different spatial structures of the velocity shear regions in the two models. The maximum dimensionless azimuthal wave number at the position  $x = LR_E$  in the equatorial plane is given by

$$m_{max} = xk_{max} \quad (5)$$

where  $R_E$  is the Earth’s radius.

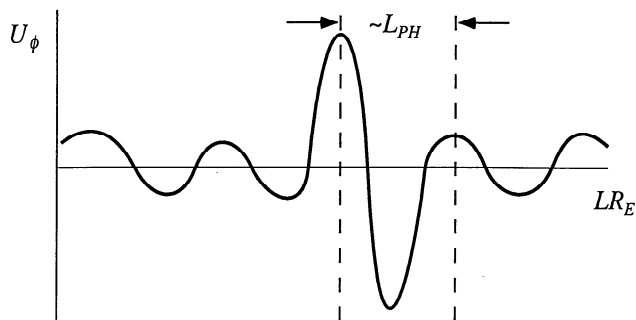
For an FLR the velocity  $V_0$  is sinusoidal in time with a frequency  $\omega_A(x_r)$  at the resonance position  $x_r$ . Growth of the K-H instability is most significant when  $V_0$  has a maximum in magnitude, which occurs twice during a cycle. Between these two maxima  $V_0$  passes through zero, so the velocity shear is only important for, say, a quarter of a cycle around the maximum. Therefore we take the normalized maximum K-H growth rate to be

$$\Gamma_{KH} = \gamma_{max}/4\omega_A. \quad (6)$$

**Table 1.** Field Line Resonance Parameters

$L$	$f$	$\omega_A$	$\omega'_A$	$V_{0M}$
11.0	1.3	$8.2 \times 10^{-3}$	$2.6 \times 10^{-10}$	$10^5$
9.3	1.9	$1.2 \times 10^{-2}$	$4.6 \times 10^{-10}$	$8 \times 10^4$
8.1	2.6	$1.6 \times 10^{-2}$	$7.2 \times 10^{-10}$	$6 \times 10^4$

Note that  $f$  is in millihertz,  $\omega_A$  in s $^{-1}$ ,  $\omega'_A$  in m $^{-1}$ s $^{-1}$ , and  $V_{0M}$  in meters per second.



**Figure 2.** Schematic of the variation of azimuthal plasma velocity  $U_\phi$  with radial distance in the equatorial plane. Phase mixing develops a local scale length  $L_{PH}$  given by (2) which continually decreases with time. This implies an increasing velocity shear with time having an extent of order  $L_{PH}$ .

Growth of K-H waves during the quarter cycle of large velocity shear will be significant if  $\Gamma_{KH}$  is comparable to or greater than unity.

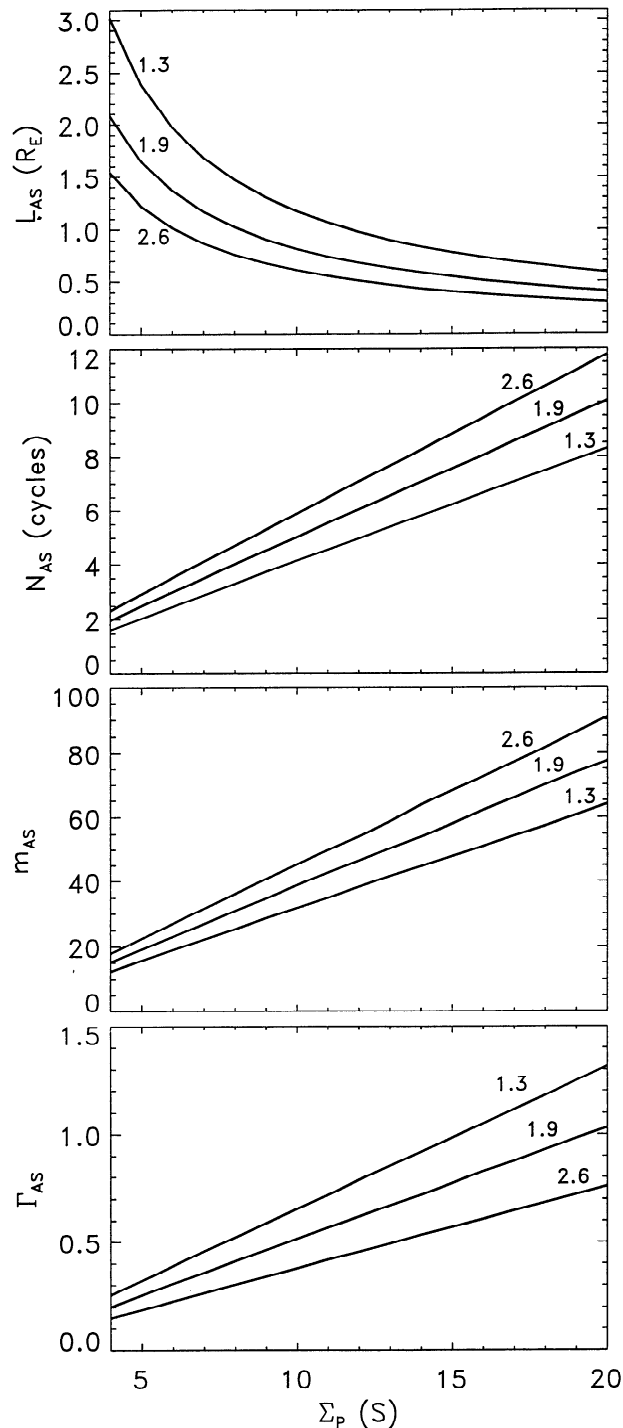
As discussed by Mann *et al.* [1995], ionospheric dissipation limits the finest scales that can be achieved for significant wave amplitude. If we assume that after two  $e$ -folding ionospheric damping times the asymptotic phase-mixing length  $L_{AS}$  is reached, then  $L_{AS}$  is given by

$$L_{AS} \sim \pi \gamma_I / \omega'_A \quad (7)$$

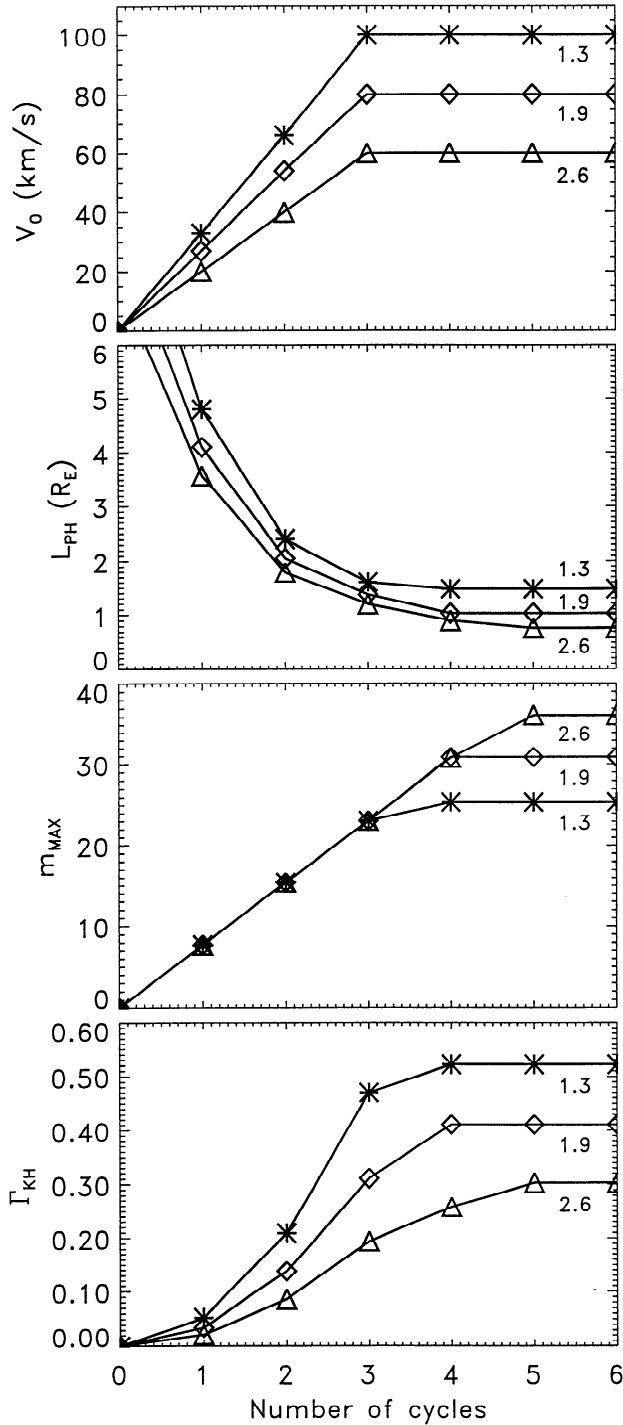
where  $\gamma_I$  is the ionospheric damping rate of an undriven standing Alfvén wave. For ionospheric Pedersen conductance  $\Sigma_P$ , we determine  $\gamma_I$  for fundamental standing Alfvén modes from the dipole model of Allan and Knox [1979]. Figure 3 gives the asymptotic values obtained for the three FLRs in Table 1 for a realistic range of  $\Sigma_P$ . The top panel shows how  $L_{AS}$  decreases with increasing  $\Sigma_P$ . The second panel shows the increasing number of wave cycles  $N_{AS}$  that is required to reach  $L_{AS}$ . The bottom two panels show the dimensionless wavenumber  $m_{AS}$  and normalized K-H growth rate  $\Gamma_{AS}$  derived from (5) and (6) of the fastest growing mode of the asymptotic state. Over the typical range  $\Sigma_P = 4$  to 10 S, the range of wavenumber is about 12 to 40, consistent with the observations. Normalized growth rates are in the range 0.15 to 0.6, sufficient to generate significant K-H waves, but not large enough to disrupt the FLRs. The number of FLR cycles required to reach the asymptotic values is in the range 2 to 5, shorter than most FLR lifetimes. Note that the growth rates are likely to be upper limits, as we have not included the stabilizing effect of field line bending [e.g., Miura, 1996].

Figure 3 relates to the asymptotic state. We now consider the temporal development of the three FLRs, the parameters associated with them, and how they tend to the asymptotic state. The ionospheric Pedersen conductance is chosen to be 8 S in each case. For simplicity we take the FLR amplitudes to increase linearly from 0 to  $V_{OM}$  over 3 cycles (typical of data) and remain

at those values for the next three cycles, as shown in the top panel of Figure 4. (We do not concern ourselves with the decay phase as we are interested in the stability of the FLRs.) We calculate the derived parameters until the asymptotic number of cycles (Figure 3) is reached in each case, then “freeze” the parameters at



**Figure 3.** Asymptotic parameters for the three FLRs with frequencies  $f = 1.3, 1.9$ , and  $2.6$  mHz versus  $\Sigma_P$ . Panels from top show the asymptotic phase-mixing length  $L_{AS}$ ; the number of FLR cycles  $N_{AS}$  required to reach  $L_{AS}$ ; the asymptotic azimuthal wavenumber  $m_{AS}$ ; and the asymptotic normalized maximum K-H growth rate  $\Gamma_{AS}$ .



**Figure 4.** Temporal development of parameters over 6 FLR cycles for  $\Sigma_P = 8$  S. Panels from top show the maximum  $\mathbf{E} \times \mathbf{B}$  drift velocity  $V_0$  of the FLRs; the phase-mixing length  $L_{PH}$ ; the maximum azimuthal wavenumber  $m_{max}$ ; and the normalized maximum K-H growth rate  $\Gamma_{KH}$ .

the asymptotic values. The second panel of Figure 4 shows how  $L_{PH}$  decreases with time for each FLR until leveling off at the asymptotic values for  $\Sigma_P = 8$  S. In the third panel of Figure 4  $m_{max}$  increases with time in the same way for the three FLRs, but levels off at different  $m_{max}$  values. The highest frequency FLR (at

the smallest  $L$  value) reaches  $m_{max} = 36$ , but takes the longest time to do so. The lowest frequency FLR (at the largest  $L$  value) only reaches  $m_{max} = 25$ , but achieves its maximum more quickly. The bottom panel of Figure 4 shows that the lowest frequency (outermost) FLR develops a larger normalized growth rate more quickly than the others.

Note that in the frame of a small- $m$  FLR (which is effectively at rest in the magnetosphere) the K-H instability is a zero-frequency instability, that is the growing perturbations are oscillatory in space but not in time [e.g., Walker, 1981].

### Amplification by Drifting Ions

In the previous section we showed that model small- $m$  FLRs similar to those observed by FSSG should be unstable to the nonlinear K-H instability. Growth within a quarter-cycle of an FLR can be significant, but not sufficient to disrupt the FLR. Within a few cycles, an FLR at a particular position should develop a dominant K-H wave with a single azimuthal wavenumber in the range  $m \sim 10$  to 80 depending on the ionospheric Pedersen conductance at the resonant field line footprint.

The K-H wave is zero frequency with an oscillatory spatial structure in azimuth within the FLR region, and should have nonnegligible amplitude. However, the wave would most likely be too small to observe directly with available instruments. As discussed by FSSG, the characteristics of the large- $m$  waves they observed suggest strongly that the waves were amplified by an energetic particle instability. The westward phase velocities and low frequencies (consistent with fundamental field-aligned standing modes) suggest to us that drift instability of energetic ions is the most likely mechanism, although we do not exclude other possibilities.

We propose that the dominant large- $m$  K-H wave provides a seed for the growth of an ion-driven instability. A simple generic dispersion relation for a drift mode is of the form

$$\omega = m\omega_d \quad (8)$$

where  $\omega_d$  is the drift frequency (e.g., gradient-curvature drift) of the resonant ion population. The K-H wave provides the  $m$  in (8). It seems likely that the  $\omega$  in (8) is determined by the development of a standing wave structure along the magnetic field, as this allows wave energy at the standing wave frequency to remain localized on the field line. With  $\omega$  determined the drift wave will grow provided energy is available in the part of the unstable ion distribution with drift frequency  $\omega_d$  as required by (8). Since the drift wave has moderately large  $m$ , the standing wave frequency should be closer to the guided poloidal mode frequency [Cummings *et al.*, 1969] than the toroidal mode frequency of the original small- $m$  FLR. For fundamental standing waves this could mean a small but noticeable difference between the small- $m$  and large- $m$  frequencies. Walker and Pekrides [1996] showed that standing wave frequencies for large  $m$  can depend on the relative variations of Alfvén and sound speeds along the field line.

However, we feel that, in the absence of detailed results to the contrary, the large- $m$  FLRs should have similar but not necessarily identical frequencies to the small- $m$  FLRs from which they develop. For less than optimum observational conditions, the large- $m$  FLRs should appear to have the same frequency as the small- $m$  FLRs. It is possible that for optimum observational conditions a frequency difference could be detectable.

The purpose of this short paper is to suggest the possibility that the nonlinear K-H instability occurring within large-amplitude FLRs can provide large- $m$  seed waves for amplification by unstable ion distributions. The exact mechanism by which the waves are coupled to and driven by the unstable ions is a major topic in itself. There are many theories in the literature, none giving a completely satisfactory explanation of the process. We therefore do not go into further detail, but a short description of work in this area can be found in section 2.3.2 of *Allan and Poulter* [1992].

Given the possibility that a westward drifting unstable ion distribution can amplify a seed K-H wave within an FLR region, it seems natural that the large- $m$  wave would reach a relatively large amplitude toward the westward edge of the small- $m$  FLR, where it could have a westward group velocity of the order of the drift velocity of the amplifying ions. We might then speculate that the large- $m$  wave could emerge from the small- $m$  FLR region and continue westward until damping processes overcame amplification, and the wave decayed. The large- $m$  wave might be observable at the westward edge of the small- $m$  FLR region, or it might continue to grow to observable size if the associated ion distribution were still unstable. It would therefore be possible to see large- $m$  FLRs westward of (but on the same  $L$  shell as) the originating small- $m$  FLR region.

## Summary and Conclusions

ULF waves with large  $m$  have been observed on similar  $L$  shells and with similar frequencies to apparently typical small- $m$  FLRs. We have extended the waveguide interpretation proposed by *Fenrich et al.* [1995] to explain these properties by invoking the nonlinear Kelvin-Helmholtz instability of the FLRs. We assume the small- $m$  FLRs are driven by small- $m$  waveguide modes on the magnetospheric flanks. Three model FLRs are considered with typical  $L$  values, frequencies, and amplitudes.

By applying the phase-mixing results of *Browning and Priest* [1984] and *Mann et al.* [1995], we show that after a few cycles the small- $m$  FLRs should each develop a single dominant zero-frequency Kelvin-Helmholtz wave component with  $m$  in the range of about 10 to 80. The asymptotic value of  $m$  depends on a balance between the effects of phase mixing and ionospheric dissipation.

Growth rates of the Kelvin-Helmholtz instability within each half cycle of the FLR are not large enough to disrupt the small- $m$  FLR, but are large enough to generate a structure within the FLR region which could act as a large- $m$  seed wave. We propose that westward drifting unstable ion distributions amplify this seed wave to

generate a large- $m$  ion drift wave. The frequency of the drift wave is determined by a field aligned standing wave structure, and should be very similar but not necessarily identical to the frequency of the original FLR. The drift wave should grow as the ions pass westward through the small- $m$  FLR region. We would therefore expect the drift wave to become visible on the westward edge of the small- $m$  FLR region, or possibly west of that edge if the ion distribution continues to be unstable.

In particular our model makes the following predictions, which are in excellent agreement with observations.

1. The magnetosheath flow has a stagnation point at the sub-solar point. As the flow is accelerated around the flanks the magnetopause becomes K-H unstable, and will excite waveguide modes in the magnetospheric flanks. The dispersive waveguide model [*Wright, 1994; Wright and Rickard, 1995*] predicts that small- $m$  FLRs should be established on the flanks, in accord with the observations of FSSG. (Note that pressure pulse excitation could establish FLRs in the subsolar region; our mechanism should also operate in such cases.)

2. The FLRs produced by the waveguide model have small  $m$  ( $< 10$ ), quantized frequencies, and antisunward azimuthal phase velocities of the order of the sheath flow speed [e.g., *Wright and Rickard, 1995; Ziesolleck and McDiarmid, 1994; FSSG*].

3. The small- $m$  FLRs will be unstable to a large- $m$  ( $10 < m < 80$ ) K-H instability, but not so unstable that they will be completely disrupted. The small- $m$  FLRs should be long-lived and easy to observe.

4. Unstable drifting ions may interact with the K-H seeds. This will produce observable large- $m$  waves near to or beyond the westward edges of the small- $m$  FLRs.

5. The large- $m$  FLRs will be on the same  $L$  shells as the small- $m$  FLRs, and will appear to have similar frequencies.

Much of our argument is qualitative, but we feel that the complete scenario is consistent with observation. Further more quantitative development should be fruitful. In particular, a detailed treatment of the amplification of the Kelvin-Helmholtz seed wave by the unstable drifting ions may clarify the surprising equatorward phase propagation of the large- $m$  waves observed by FSSG and others.

**Acknowledgments.** W.A. acknowledges support from the New Zealand Foundation for Research, Science and Technology (FRST) through contract CO1627. A.N.W. is supported through a UK PPARC Advanced Fellowship, and is grateful to PPARC and FRST for funding his visit to NIWA, New Zealand.

The Editor thanks A. David M. Walker and another referee for their assistance in evaluating this paper.

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(Received December 20, 1996; revised May 13, 1997; accepted May 14, 1997.)