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Time-limited excitation of damped field-line resonances: Implications for satellite observations

D. R. McDiarmid

Herzberg Institute of Astrophysics, National Research Council of Canada, Ottawa, Ontario, Canada

A. N. Wright

Department of Mathematical and Computational Sciences, University of St. Andrews, St. Andrews Fife, Scotland

W. Allan

National Institute of Water and Atmospheric Research, Wellington, New Zealand

Abstract. It is common to think of resonance width in steady state terms, but there are two important factors which make this approach inappropriate for field-line resonance (FLR) pulsations in the magnetosphere. First of all, the excitation is short-lived, i.e., decidedly not monochromatic. Second, for excitations of interest, FLR widths diminish with time [*Mann et al.*, 1995], and the ionospheric damping of these resonances in concert with their short-lived excitation ensures that they are time-limited. The widths most likely to be observed are those corresponding to a time near that of the maximum amplitude of the pulsation. As a consequence, observed FLR widths are greater, sometimes significantly greater, than the steady state equivalent (i.e., the Q width). Each FLR oscillation is the sum of two components; one is an oscillation at the resonant frequency, and the other, which is the transient portion of the response, is at the natural resonant frequency of each flux tube. The latter frequency varies with L. Moreover, as the excitation becomes more short-lived, the shorter is the duration of the FLR in a lossy magnetosphere/ionosphere, and the greater the influence of the transient. The purpose of this paper is to examine the possibility that a random or quasi-random ensemble of short-lived excitations of the magnetospheric waveguide over the Pc3 - 5 range might yield FLR widths sufficiently large in practice for the pulsation signature to be similar to that seen in Active Magnetospheric Particle Tracer Explorers/Charge Composition Explorer (AMPTE/CCE) satellite pulsation data. The latter signature appears to be consistent with each flux tube ringing at several of its natural resonance harmonics. It is found that under some realistic conditions the ensemble of excitations can produce FLR spectral widths which are contiguous or overlap in L, suggesting the possibility of an FLR explanation of AMPTE/CCE pulsation observations.

1. Introduction

Recently, *Allan et al.* [1997] explored the mystery which is the dichotomy of the implications of ground-based pulsation observations versus those of the Active Magnetospheric Particle Tracer Explorers/Charge Composition Explorer (AMPTE/CCE) satellite. The former are naturally explained as field-line resonances (FLR) in which a latitudinal band of flux tubes are all excited at one frequency concurrent with each of these flux tubes also oscillating at its natural resonant frequency [e.g., *Samson and Rostoker*, 1972; *Poulter*, 1982; *Allan et al.* ,1985; *Ziesolleck et al.*, 1996]. These are the driven and transient responses respectively. The AMPTE/CCE data, however, seem best explained in terms of the continuum of closed magnetic field lines oscillating in the shear mode at their own specific resonant frequencies [e.g., *Takahashi and McPherron*, 1984; *Engebretson et al.*, 1986; *Anderson et al.*, 1990]. These frequencies correspond to those of the transient portion of the FLR. *Allan et al.* [1997] simulated the AMPTE/CCE signature of typical Pc5 quasi-monochromatic latitude-constrained FLR pulsations commonly observed by ground-based instrumentation, such as magnetometers and radars. They found that the AMPTE/CCE signature of such events is not clearly distinct from the signature normally obtained from AMPTE/CCE data. Their signature spanned a nontrivial interval of L (i.e., orbit time) owing to the finite radial width of the FLR.

Ziesolleck et al. [1997] recently simulated a magnetospheric waveguide in which the field-line frequencies decreased with L (or latitude), as expected for the magnetosphere. The location and frequency of the FLRs were determined using the approach described by *Wright* [1994]. The result (Ziesolleck et al.'s Figure 16) suggested that, for

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any given field-line harmonic, j, the FLR position of the radial harmonics, k, could be sufficiently close to each other in L so that the latitudinal bandwidths of individual FLRs might be closely adjacent or even overlap and, in thus merging together, possibly produce an AMPTE/CCE signature similar to that which has been observed. It is the purpose of this paper to examine this possibility.

It is important to be clear about the sort of resonant behavior we are dealing with. The resonators in this case are the geomagnetic field lines in shear Alfvén wave motion. Resonances are commonly thought of in terms of their driven response (without the transient) to steady state or monochromatic excitation. However, pulsation resonances often experience short duration excitation. Also ideal resonators are lossless, whereas pulsation resonators are damped, often significantly. Thinking of the behavior of pulsation resonators in terms of steady state solutions can be seriously misleading; time-dependent solutions are required.

Allan et al. [1997] noted that the meaning of the resonance concept is clear for an ideal (lossless) resonator driven for an infinite time by steady or monochromatic excitation. Here the bandwidths of both the steady state excitation ($\Delta\omega_d$) and the resonant response ($\Delta\omega_A$) are zero. Finite amplitude excitation produces a response which grows toward infinite amplitude. For a system containing resonators having a continuum of resonant frequencies, as does the magnetosphere in the Alfvén (shear) mode ($\omega = f(L)$), at least one field line is resonant at any given frequency within the continuum band, and, if the system is lossless, the classic resonance singularity exists for any excitation whose frequency is within the continuum.

However, in the magnetosphere-ionosphere system, excitation duration is finite ($\Delta\omega_d > 0$) and resonances are damped ($\Delta\omega_A > 0$). Thus the response amplitude is never infinite. The question then becomes: On which field lines will large-amplitude Alfvén waves (LAWs) be excited by a monochromatic or quasi-monochromatic driver? When the excitation is of finite duration, the excitation bandwidth is nonzero, and when the resonator is lossy, the response bandwidth is nonzero in the steady state. *Allan et al.* [1997] discussed four situations [($\Delta\omega_d = 0$, $\Delta\omega_A = 0$), ($\Delta\omega_d = 0$, $\Delta\omega_A > 0$), ($\Delta\omega_d > 0$, $\Delta\omega_A = 0$), and ($\Delta\omega_d > 0$, $\Delta\omega_A > 0$)], although only the last is physically meaningful.

We have discussed above the first of the *Allan et al.* [1997] possibilities. For ($\Delta\omega_d = 0$; $\Delta\omega_A > 0$), there is no singularity, and, as a consequence, monochromatic excitation produces LAWs over a contiguous set of latitudes or L shells centered on that which is resonant at the excitation frequency, provided of course that the excitation frequency lies within the Alfvén wave frequency band of the geomagnetic field lines. Note that $\Delta\omega_d = 0$ only when the steady state is achieved. The response occurs between L_1 and L_2 where these L shells have resonant frequencies of ($\omega_d + \Delta\omega_A/2$) and ($\omega_d - \Delta\omega_A/2$), respectively. This situation is also commonly called a resonance, and the resonance width, $\Delta\omega_A$, is not, by definition, a function of time but is a function of Σ_P, the height-integrated Pedersen conductivity. The response, however, is monochromatic; the transient having decayed leaving only the oscillation at ω_d. The commonly known resonance Q factor is the ratio $\omega_d/\Delta\omega_A$.

The ($\Delta\omega_d > 0$, $\Delta\omega_A = 0$) case corresponds to an ideal system driven coherently for a finite time or incoherently for any length of time. LAWs occur between L_1 and L_2 corresponding to ($\Omega + \Delta\omega_d/2$) and ($\Omega - \Delta\omega_d/2$), where Ω is ω_d for a coherent driver and is the most probable value of ω_d for the incoherent driver. In the former case (finite interval of coherent excitation), $\Delta\omega_d$ is time-dependent. The response oscillates at both the transient ($\omega_A(L)$) and driver (ω_d) frequencies. In other words, the overall response is spectrally structured.

The most complicated situation, and the one which is physically realistic, is that of time-limited or broadband excitation of a dissipative resonance; that is, both $\Delta\omega_d$ and $\Delta\omega_A$ are nonzero. This is the situation which applies throughout this paper. Here one might expect the LAWs to be excited between ω_l and ω_u where $\omega_l \approx \omega_d - (\Delta\omega_d + \Delta\omega_A)/2$, $\omega_u \approx \omega_d + (\Delta\omega_d + \Delta\omega_A)/2$, and ω_d is the (most probable) driver or excitation frequency, although, as we will show, the situation is actually more interesting than this. Again the response contains both transient and driver frequencies. Since $\Delta\omega_d$ decreases as the duration of coherent excitation increases, the resonance spectral width, ($\omega_u - \omega_l$), also diminishes with time. However, the spectral width one should expect to observe is weighted toward that for the time of maximum resonant response. It is clearly no longer appropriate to refer to response bandwidth in terms of Q as this is a steady state parameter. The difference ($\omega_u - \omega_l$) can be significant and normally corresponds to a broad latitudinal bandwidth via the $\omega = f(L)$ relationship. Nevertheless, the response will be seen by ground instrumentation as occurring within a confined latitude interval and will have the properties of a resonance (FLR), which is to say, a peaking of the shear mode amplitude in L or latitude and a phase change of about 180° through that peak. This example of LAWs has commonly been called a resonance, and we will call it such throughout this paper.

It will be shown that for a subset of this most realistic case, the "resonance" condition of equality between the complex excitation and resonator frequencies [e.g., *Allan et al.*, 1997] can occur. Under this condition, the shear mode solution for the resonant field line contains a singularity very similar to that which appears in the ideal case. The amplitude of the response, however, remains finite. The limited duration of the excitation limits the amount of energy which is coupled into the oscillation. Mathematically, the singularity is balanced by the remainder of the solution going to zero. Dissipation extinguishes the response as it narrows to the singularity state.

The meaning of the driver and resonance spectral widths must be defined. The full width at half maximum (FWHM) of the spectral (or latitudinal) distribution is often used. One must be clear, however, as to whether it is the power or the amplitude spectral distribution. Half power is commonly used; it corresponds to $1/\sqrt{2}$ width for the corresponding amplitude. This is the definition used herein.

To reiterate, a key point in the case of decaying short-duration excitation of damped FLRs is that the asymptotic spectral width of the FLR response is not the important width. Such widths are obtained only when the FLR response has damped to an inconsequential amplitude. The spectral width of importance is that corresponding to the interval of greatest FLR amplitude as that is the interval to which observations will be weighted. This width is greater, sometimes much greater, than the asymptotic value. The question then arises as to whether a collection of FLRs with significantly but not unreasonably large damping rates can have spectral bandwidths which overlap in frequency and thus overlap in L when generated by broadband quasi-random excitation of the magnetospheric waveguide by sources of short duration. If they can, a potential explanation of the AMPTE/CCE signatures presents itself.

The paper is structured as follows: A waveguide having a realistic frequency versus latitude structure is constructed in section 2, and the location and frequency of the FLRs corresponding to a range of j (field-line harmonic) and k (radial harmonic) are determined; in section 3, the realistic magnitude of the wave reflection coefficient at the ionosphere-magnetosphere boundary is discussed; section 4 addresses the latitudinal bandwidth ($\omega_u(L)$ - $\omega_l(L)$) and how it varies with j and k; section 5 discusses the implications of these results to the AMPTE/CCE observations; and section 6 summarizes the main points of the paper. The results are complementary with those recently presented by *Rickard and Wright* [1995], *Mann* [1997] and *Allan et al.* [1997].

2. Waveguide Model and its FLR ω versus L Plot

We employ the rectangular waveguide model of *Wright* [1994]. The dimensional parameters are defined by Wright where the coordinates have the following meaning: x is the radial or L direction, y is the azimuthal direction, and z is the field-line direction. Therefore the background or geomagnetic field is in the z direction and is constant with x as required. The strategy is to construct a cold plasma mass density variation with x, which produces a field-line frequency versus x dependence approximating that of the Earth's dayside magnetosphere. Our construction is based on the results of *Poulter et al.* [1984] who transformed pulsation versus latitude data obtained from radar observations into plasma mass density versus L. They assumed an r^{-4} variation of the mass density along the assumed dipole geomagnetic field lines after correcting the observed periods for the divergence of the geomagnetic field from a dipole using the results of *Singer et al.* [1981]. Having thus transformed their data, they averaged the pre-geomagnetic and post-geomagnetic noon data to obtain 194,372 $L^{-4.75}$ and 15,039 $L^{-3.08}$ amu cm^{-3}, respectively. The availability of the Poulter et al. results precluded the necessity of a significant preliminary effort to obtain the equivalent from the Canadian Auroral Network for the OPEN Unified Study (CANOPUS) data through, for instance, the use of cross-phase and similar techniques [e.g., *Baransky et al.*, 1985; *Waters et al.*, 1991].

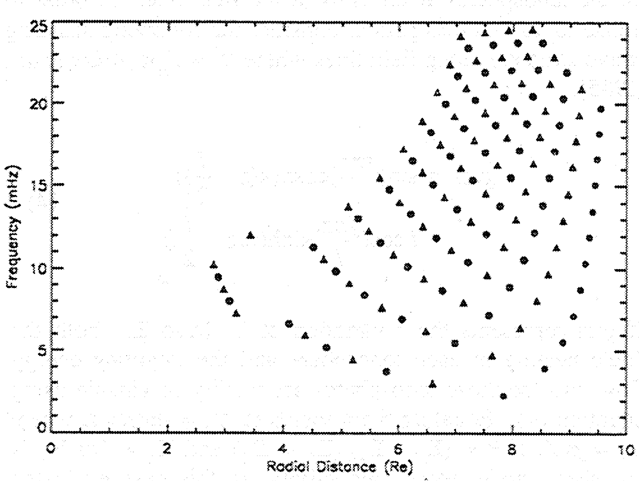

Figure 1. The waveguide cutoff frequencies and associated shear mode resonance positions as a function of the field-line harmonic number (j) and radial harmonic number (k) for the prenoon magnetosphere. The progression upward in frequency and outward in radial resonance position of a given symbol corresponds to j increasing from 1. The progression upward in frequency and inward in resonance position corresponds to k increasing from 1 with j constant.

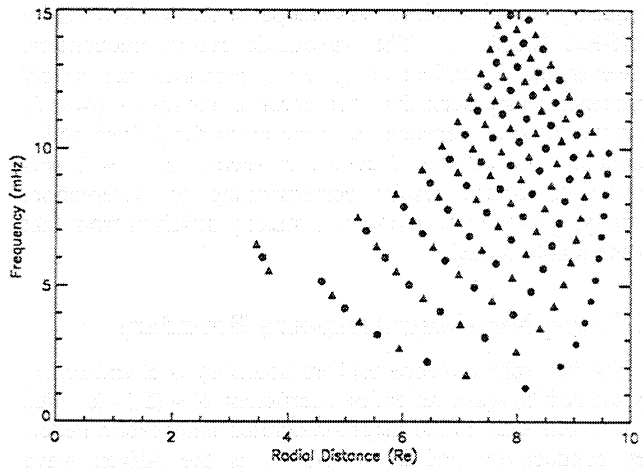

Figure 2. The same as Figure 1, except for the postnoon magnetosphere.

We have employed these results and have added a plasmapause and outer plasmasphere. The plasmapause is assumed to be 0.5 R_e thick within which the mass density varies as L^{-11}, as suggested by a specific observation of *Poulter et al.* [1984]. Inward of the plasmapause, the mass density is assumed to vary as L^{-3}. The plasmapause is placed at $3.5 < L < 4.0$ and $4.0 < L < 4.5$ for the prenoon and postnoon sectors, respectively.

The above two mass density profiles were converted to corresponding profiles in the rectangular waveguide model such as to retain the original Alfvén wave period versus L variation. The parameters x and L are synonymous for the waveguide. The transformed profiles were converted into V_A profiles, where V_A is the Alfvén velocity. Given these profiles, the WKB method of *Wright* [1994, equation (24)] was used to obtain the waveguide cutoff frequencies for each of (j,k), which together define a waveguide mode, where k is the radial harmonic and j is the field-line harmonic. These cutoff frequencies closely approximate the optimum frequency of excitation of shear mode oscillations by the waveguide mode [*Wright*, 1994].

A computer program was written to determine these cutoff frequencies, and the results for $j,k < 13$ are shown in Figures 1 and 2. The shapes of these two plots are similar; the only significant difference is the diminishing of the frequencies in going from the prenoon plot to the postnoon. This is as expected since it is the consequence of flux tube filling during their dayside traversal. The radial position of any given resonance (j,k) moves to larger L from prenoon to postnoon. This shift is small for $k = 1$ and j large, but more significant for larger k.

The transformation of the mass density profiles from dipole to box model was done such that the natural resonant frequency variation with L was independent of the box model field-line length, l_0. The waveguide cut-off frequencies, however, are dependent on l_0. As l_0 increases, the cut-off frequencies vary such that their location moves to lower L, and the distance between them increases for j fixed and k variable. As will be discussed in section 5, $l_0 = 2$ was chosen to obtain results corresponding to observation. Clearly, l_0 in the box model is distinctly different from that in the dipole model.

3. Ionosphere-Magnetosphere Boundary

The ionosphere-magnetosphere boundary is characterized by the Alfvén wave reflection coefficient, $R = (\Sigma_P - \Sigma_A)/(\Sigma_P + \Sigma_A)$, where Σ_P is the height-integrated ionospheric Pedersen conductivity and $\Sigma_A = 1/\mu_0 V_A$ is the Alfvén wave conductivity, in this case evaluated adjacent to the ionosphere. (A more sophisticated reflection process [e.g., *Yoshikawa and Itonaga*, 1996; *Buchert and Budnik*, 1997] could have been employed, but this would have added greatly to the complexity of the paper without significantly changing the results.) Using the parameters adopted above for the magnetosphere model, Σ_A for a dipole geomagnetic field can be readily determined. Values of Σ_A of about 0.2 and 0.3 mho are obtained for the prenoon and postnoon sectors, respectively. The value of Σ_P, which depends on solar illumination and energetic particle precipitation, is more problematic. Here we use the results of *Wallis and Budzinski* [1981] who suggest that $5 < \Sigma_P < 10$ mho are appropriate for most of the dayside at most levels of disturbance. Ionospheric reflection coefficients can then be expected to lie close to the interval of $0.87 < R < 0.97$. The appropriate value of R will, of course, depend on the time of day and the level of magnetospheric disturbance.

4. Resonance Widths

Pulsation wave trains are of quite limited duration, particularly at lower frequencies. Observation shows that the number of periods in pulsation wave trains diminishes with frequency to be typically < 10 for Pc5 pulsations. The length of the wave trains is observed to be larger, in terms of the number of periods, at higher frequencies. This factor is important to the outcome of this investigation. *Mann et al.* [1995] have shown that the dynamic shear mode or FLR resonance width of an ideal resonator corresponds to that of the driver signal. They point out that this width will appear large to a satellite encountering a resonance region soon after the resonance is excited and much narrower if the encounter is long after the excitation. When the resonator has dissipation, this width is expected to increase. If a damped FLR is excited by a driver of limited duration, the response spectral bandwidth might be expected to be something like $(\Delta\omega_d(x,t_m) + \Delta\omega_A(x))$, where t_m is the time of maximum shear mode response. At this time, the width in L over which the FLR exists can be significantly larger than what would be observed in the steady state.

We have examined the time development of $\Delta\omega_d$ for the following excitation function:

$$f_d(t) = \begin{cases} 0 & t < 0 \\ A\, t\, e^{-\alpha t} \sin \omega_d t & 0 \le t \le t' \\ 0 & t > t' \end{cases} \qquad (1)$$

This function was chosen to mimic an excitation having a gradual onset. The Fourier transform of this function is

$$F_d(\omega,t') = \frac{A}{4\pi i}\left[\frac{(1+\nu_+ t')e^{-\nu_+ t'} - 1}{\nu_+^2} - \frac{(1+\nu_- t')e^{-\nu_- t'} - 1}{\nu_-^2}\right] \qquad (2)$$

where

$$\nu_+ = \alpha + i(\omega + \omega_d)$$
$$\nu_- = \alpha + i(\omega - \omega_d)$$

The power spectral density, which is a function of time, is given by $F(\omega,t')F(\omega,t')^*$, where the asterisk denotes complex conjugate.

With waveguide excitation of FLRs, the exponential decay of this driving function represents a temporal diminution of compressional mode energy at any given azimuth along the magnetospheric waveguide. The sources of this diminution or loss include (1) energy propagation either down the waveguide away from the field lines of interest or through the waveguide boundary into the polar cap or out of the magnetosphere through an imperfectly reflecting magnetopause, (2) some loss directly to the ionosphere although this is likely to be minimal [*Kivelson and Southwood*, 1988], and (3) loss to the shear or Alfvén mode through excitation of the FLR.

Following *Inhester* [1987] and *Krauklis and Orr* [1994], we represent the shear mode oscillation using

$$\frac{\partial^2 \xi_y}{\partial t^2} + 2\omega_i \frac{\partial \xi_y}{\partial t} + \omega_r^2 \xi_y = f_d \qquad (3)$$

where ξ_y represents the y component of the plasma displacement and ω_i represents energy dissipation via Joule heating in the ionospheres at the ends of the field line. In order to relate ω_i to that dissipation, consider the following standing wave structure along field lines where $\omega_i = V_A \kappa$ [*Allan et al.*, 1985].

$$g(z) = \sin\left(\frac{j\pi z}{l}\right)\cosh\left(\kappa\left(z - \frac{l}{2}\right)\right) + i\cos\left(\frac{j\pi z}{l}\right)\sinh\left(\kappa\left(z - \frac{l}{2}\right)\right) \qquad (4)$$

If $g(z)$ represents the z variation of E_x (also ξ_y), both the Joule heating in each ionosphere and the Poynting energy flow into the those ionospheres are readily obtainable using equation (4). Equating them yields $\omega_i/\omega_r = -\ln(R)/j\pi$ where $\omega_r = j\pi V_A/l$, $R = (\Sigma_P - \Sigma_A)/(\Sigma_P + \Sigma_A)$ and $\Sigma_A = 1/\mu_0 V_A$ is the shear mode wave conductivity, in this case evaluated adjacent to the ionosphere boundary. Note that ω_i/ω_r varies as $1/j$, that is, the damping rate decreases as the field-line harmonic number increases. Since the z dependence of ξ_y is the same as that for E_x, we write $\xi_y(x,z,t) = g(z)\Psi_y(x,t)$.

Various methods can be used to solve equation (3); we used the Laplace transform method. The initial conditions are $\xi_y = \partial\xi_y/\partial t = 0$. Note that here t' in equation (1) is ∞. The solution is

$$\Psi_y(x,t) = \frac{2A}{\omega_d\omega_r' D(\omega_r',\omega_d)}\left[\frac{t}{2}e^{-\alpha t}\sin(\omega_d t - \theta_1) - e^{-\alpha t}\left[\frac{\sqrt{1+\left(\frac{\omega_r'}{\omega_d}\right)\delta^2}}{\omega_r' D(\omega_r',\omega_d)}\right]\cos(\omega_d t - 2\theta_1 + \theta_{\delta 1}) + e^{-\omega_i(x)t}\left[\frac{\sqrt{1+\left(\frac{\omega_d}{\omega_r'}\right)\delta^2}}{\omega_r' D(\omega_r',\omega_d)}\right]\cos(\omega_r'(x)t - 2\theta_2 + \theta_{\delta 2})\right] \tag{5}$$

where

$$\omega_r' = \omega_r\sqrt{1-(\omega_i/\omega_r)^2}$$

and

$$D = \sqrt{\delta^4 + 2\delta^2\left(\frac{\omega_d}{\omega_r'} + \frac{\omega_r'}{\omega_d}\right) + \left(\frac{\omega_d}{\omega_r'} - \frac{\omega_r'}{\omega_d}\right)^2}$$

$$\theta_1 = \tan^{-1}\left(\frac{2\delta\sqrt{\frac{\omega_d}{\omega_r'}}}{\delta^2 - \left(\frac{\omega_d}{\omega_r'} - \frac{\omega_r'}{\omega_d}\right)}\right)$$

$$\theta_2 = \tan^{-1}\left(\frac{-2\delta\sqrt{\frac{\omega_r'}{\omega_d}}}{\delta^2 + \left(\frac{\omega_d}{\omega_r'} - \frac{\omega_r'}{\omega_d}\right)}\right) \tag{6}$$

$$\theta_{\delta 1} = \tan^{-1}\left(-\delta\sqrt{\frac{\omega_r'}{\omega_d}}\right)$$

$$\theta_{\delta 2} = \tan^{-1}\left(\delta\sqrt{\frac{\omega_d}{\omega_r'}}\right)$$

$$\delta = \frac{(\omega_i(x) - \alpha)}{\sqrt{\omega_d\omega_r'(x)}}$$

The parameter A can be expected, in practice, to vary with x across the resonance but not strongly. Our assumption that it is constant will not alter the results significantly.

Note that if $\omega_i(x_0) = \alpha$, D^{-1} becomes singular when $\omega_r'(x_0) = \omega_d$, as pointed out in section 1. However, the remainder of the solution also becomes zero and L'Hopital's rule or a fresh solution under this condition gives us

$$\Psi_y(x_0,t) = \frac{A}{4\omega_d^2}t\sqrt{1+(\omega_d t)^2}e^{-\alpha t}\sin(\omega_d t - \phi) \tag{7}$$

where

$$\phi = \tan^{-1}(\omega_d t)$$

Clearly, the above solution is valid for both $\alpha = \omega_i(x_0) \neq 0$ and for $\alpha = \omega_i(x_0) = 0$. This implies that the notion that ionospheric damping will add to the bandwidth arising from $\alpha > 0$ is too simple and, for some values of ionospheric damping, is wrong, as we will now show.

Using the definition of δ above, equation (5) can be written with $\exp(-\alpha t)$ as an overall multiplicative factor. The remainder of the solution does not depend on α or ω_i directly but rather on δ. Therefore the normalized bandwidth, $\Delta\omega/\omega$, evaluated at a fixed number of excitation periods after the initiation of the excitation as a function of δ_r, the value of δ at the resonant field line, will be independent of α. Figure 3 shows this result for f_d excitation at times corresponding to 5, 10, 15, 20, 30, and 50 ω_d periods after the beginning of the excitation. The progression of these curves is from the top of the figure downward. For any given time after initiation of excitation, the spectral width or bandwidth of the FLR response varies with δ_r and is minimized at a negative value of δ_r and both the magnitude of this minimum and $|\delta_r|$ decrease toward zero with time. In the limit as $t \to \infty$, the spectral width $\to 0$ at $\delta_r = 0$ as required by the resonance condition. However, we do not in practice achieve zero bandwidth for $\alpha > 0$ because the amplitude of the response $\to 0$ as $t \to \infty$. In general, the bandwidth diminishes with time, either approaching a limiting value ($\delta_r > 0$) or reaching one ($\delta_r < 0$).

Return now to the question of what happens to $\Delta\omega/\omega$ when ionospheric loss is added to an ideal system whose

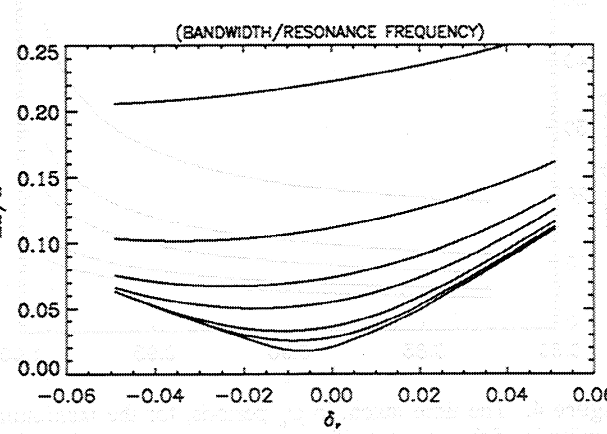

Figure 3. The frequency bandwidth of equation (5) at 5, 10, 15, 20, 30, 40, and 50 ω_d periods after the initiation of the response as a function of δ_r which is the value of δ at the resonance. The curves shift downward from the top of the plot as the number of periods increases.

excitation has $\alpha > 0$. In this case, δ_r is initially < 0. As ionospheric loss is added, ω_i increases from zero, and therefore δ_r increases toward zero and then beyond (i.e., to the right in Figure 3). The consequence of this change is that for lower numbers of ω_d periods (upper curves in Figure 3), $\Delta\omega/\omega$ increases with δ_r as expected but at times corresponding to a larger number of ω_d periods (lower curves), it might initially decrease before increasing. In short, increasing ω_i does not always imply an increase in spectral bandwidth as suggested by *Allan et al.* [1997].

However, what sort of $\Delta\omega/\omega$ will one see in practice? A physical event is most likely to be seen when its amplitude is greatest. Therefore the relevant values of $\Delta\omega/\omega$ are those corresponding to times near that of the maximum amplitude of the response. In Figure 4, we see the time taken, in ω_d periods, for the maximum amplitude of the second field-line harmonic response to be reached as a function of R for f_d excitation with $\alpha/\omega_d = 0.01, 0.02, 0.03, 0.05$, and 0.07, again with the curves progressing from top to bottom. The curves in Figure 4 move up the plot as j, the field-line harmonic, increases owing to the $1/j$ dependence of ω_i/ω_r. In each case, the maximum amplitude response time diminishes rapidly as ionospheric damping is increased from zero (i.e., R decreased from 1.0), after which it approaches a steady value. There is also a large decrease in this time as α/ω_d is increased from 0.01 to 0.03 and a further but smaller decrease as it is increased further to 0.05. In the latter case, the time of maximum response rapidly becomes < 10 ω_d periods as ionospheric damping is increased, and this corresponds to significant values of $(\Delta\omega/\omega)$ in Figure 3. This result suggests that for short f_d-like excitation bursts which have a time interval of amplitude above 0.707 of the peak value of about 5.5 periods, the observed spectral widths will be large given any reasonable ionospheric damping. This is the condition speculated about earlier in section 1.

5. Implications Regarding AMPTE/CCE Observations

With the above as background, a beginning can be made in addressing the question raised in section 1, namely what

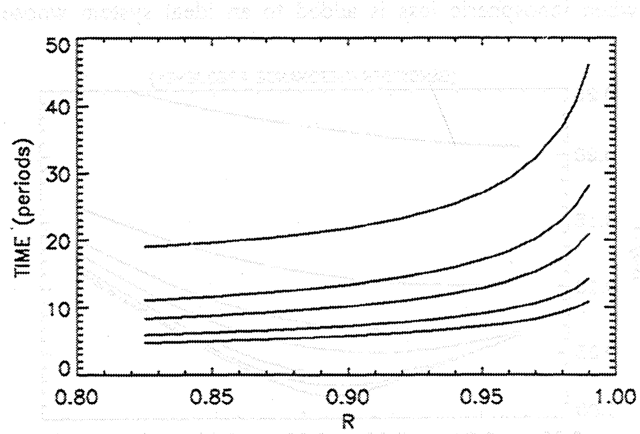

Figure 4. The time taken, in ω_d periods, for the maximum amplitude of the equation (5) response to occur as a function of the ionospheric reflection coefficient for second field-line harmonic excitation with $\alpha/\omega_d = 0.01, 0.02, 0.03, 0.05$, and 0.07. This time increases as the field-line harmonic (j) increases because $\omega_i/\omega_d \sim 1/j$. Again, the curves shift downward from the top of the plot as α/ω_d increases.

characteristics would magnetospheric FLRs reveal via an AMPTE/CCE-type analysis if created by broadband, quasi-random excitation? By broadband excitation, we mean excitation whose bandwidth not only spans any given FLR resonance frequency but also, over a short interval via an ensemble of individual excitations, an overall excitation bandwidth spanning of the order of 5-30 mHz. In other words, the excitation spectrum is wide enough and has a structure which covers a sufficient span of field-line harmonics that all the relevant waveguide modes in that frequency range are excited near their cutoff frequency. We therefore confine the mean time between the quasi-random excitations to be less than that used to compute AMPTE/CCE spectra. The question of whether such excitation is physically reasonable is, at this point, a separate issue.

If the magnetopause is perturbed over this frequency band consistent with the above description, what would the consequence be in terms of the magnetospheric waveguide model [*Wright*, 1994]? *Wright* [1994] showed that those waveguide mode signals which are effective in coupling to FLRs must have frequencies close to but just above the cutoff frequency of one of the waveguide modes. Signal energy at higher frequencies in one of these modes will propagate along the waveguide initially at increasingly larger velocity as the frequency increases and later at relatively high velocity [*Wright*, 1994]. This diminishes the contact interval that such an excitation signal will have with any given resonant flux tube. Consequently, energy coupling to a resonant flux tube is diminished. However, signals at or below the cutoff frequency cannot propagate along the waveguide. Further, the frequency must be slightly higher than the cutoff frequency for the waveguide mode to exhibit the gradients in the azimuthal direction required for energy coupling to the shear (FLR) mode. Thus there will be a frequency band over which effective coupling will occur, but it is normally narrow because the waveguide dispersion curve has a low slope ($d\omega/dk_y$) here. The frequency ω_d is that corresponding to the most effective coupling in this band.

We will represent the excitation signal for individual waveguide modes by f_d as defined above. This signal itself has a finite nonzero bandwidth because of its finite duration and its amplitude modulation. Allowing further for the existence of other excitation signals at frequencies (very) near ω_d, arising from the limited band of coupling just described, will have the effect of further broadening the spectral width of the resonant response beyond that obtained in section 4. Finally, if the excitation times are random or quasi-random as specified, there will be occurrences of overlapping excitation signals for specific waveguide modes, and this will also contribute to the frequency spectrum of the excitation [*McDiarmid and Ziesolleck*, 1996].

Two choices remain to be made in defining our excitation ensemble, namely α/ω_d and R. The latter fixes the rate of dissipation of FLR energy in the ionosphere via ω_i. This leaves α/ω_d, which describes the rate of decay of the excitation signal. Various factors contribute to α/ω_d. First, the compressional mode excitation will be of limited duration. Second, on any given flux tube, the amplitude of the compressional mode will decay if energy in that mode is (1) dissipated in the ionosphere, (2) coupled to an FLR, or (3) propagated away along the azimuthal waveguide. A remaining question is how α or α/ω_d varies from one FLR to another, i.e., as a function of the underlying waveguide mode. If α is the same for all FLRs, each of them has the same duration of excitation irrespective of frequency. Higher-frequency resonances are thereby excited for a greater number of periods. If α/ω_d is constant, irrespective of ω_d, the excitation interval, in terms of ω_d periods, is the same for all resonances. As noted above, Pc3-4 pulsations tend to endure over a larger number of periods than do Pc5 pulsations, but this is probably due primarily to the fact that $\omega_j/\omega_d \sim 1/j$. Thus constant α/ω_d appears to be consistent with observation and implies an ensemble of excitation sources, each having a duration which varies as ω_d^{-1}.

Since we can only speculate on the nature of the excitation source(s), we investigated both options. In each case, values for R and either of α or α/ω_d were chosen and the spectral width of each FLR at the time of greatest response amplitude was determined. These results were then plotted in terms of their radial equivalent, on a diagram similar to that in Figures 1 and 2. The first of these is Figure 5 and is for $R = 0.900$ with $\alpha/\omega_d = 0.05$ at all frequencies in the prenoon magnetosphere. The solid lines cease at their left-hand end because the computations were not done for radial harmonics > 12. Otherwise, the coverage would have extended further to the left. The odd harmonics were neglected because the AMPTE/CCE orbit was close to the geomagnetic equator. Except for the second field-line harmonic (lowest line), there is only one significant gap in the solid (heavy) lines. Thus, for field-line harmonics 4, 6, 8, etc., if the magnetosphere were excited consistent with the conditions specified for Figure 5 and if the amplitude of the excitation varied with frequency such as to produce resonant responses whose time derivatives all had similar amplitude, the AMPTE/CCE satellite might obtain a pulsation signature similar to what it has sometimes seen. Figure 6 shows the equivalent result for the postnoon sector. Except for the change in FLR frequencies, much the same conclusion applies although there are more gaps. Too much ought not to be made of these gaps. They do not denote the absence of oscillatory behavior but rather the presence of it at a slightly lower amplitude than in the solid line region. The solid line cutoff is the $1/\sqrt{2}$ bandwidth criterion.

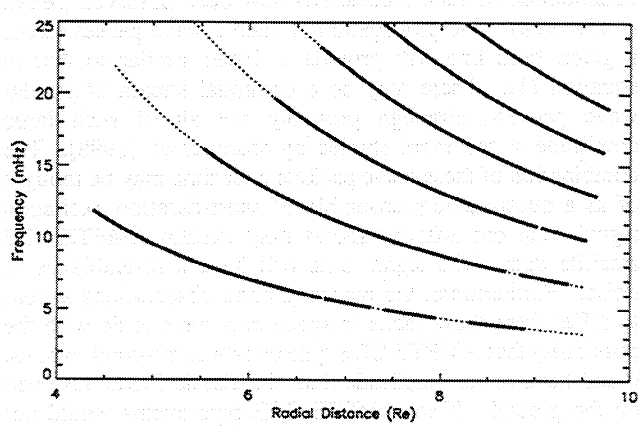

Figure 5. The heavy lines denote, for each of the first six even field-line harmonics of the prenoon magnetosphere, the extent of the radial (frequency) interval about each FLR corresponding to the spectral width of that FLR response at the time of its maximum amplitude. In most cases the intervals overlap. In this case, $R = 0.900$ and $\alpha/\omega_d = 0.05$ for all resonances.

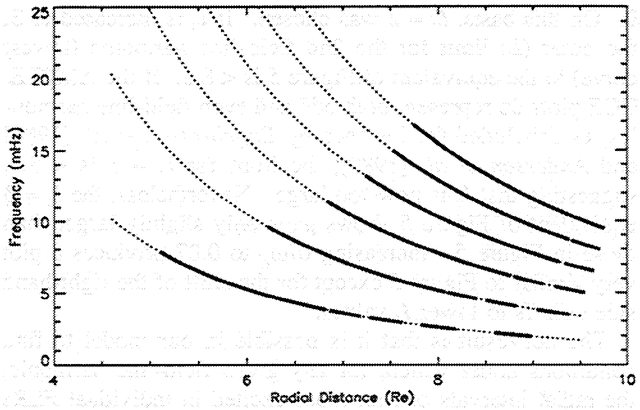

Figure 6. The same as Figure 5, except that the postnoon magnetosphere was used.

However, is the condition specified for α for Figures 5 and 6 realistic? The answer not being known, an alternate condition of α independent of frequency and set to the value corresponding to $\alpha/\omega_d = 0.05$ for the (j=1, k=1) resonance was applied. The result is shown in Figure 7. Again, the equivalent result for the postnoon sector is similar. In this case, the duration of the excitation is long enough as the frequency increases that the spectral width of the response is no longer broad when the peak amplitude of the response occurs. Consequently, there are many gaps in the lines. These gaps could be diminished if α were increased or R decreased (i.e., ionospheric loss increased), but problems remain at higher frequencies. Changing α/ω_d (j=k=1) to 0.07 or R to 0.850, for instance, diminishes the size of the gaps, particularly at the lower frequencies where some are eliminated. Nevertheless, most of the gaps remain, and, at the higher frequencies, they are not much reduced in size. However, increasing R or decreasing α or α/ω_d increases the size of the gaps under both α conditions.

Return now to the question of the choice of field-line length, l_0, in the box model. *Anderson et al.* [1989] presented results showing what they denote as fundamental toroidal mode oscillation out to the apogee of the AMPTE/CCE orbit (L = 8.8), and they extracted spectral peak values out to at least L = 8.4. *Ziesolleck and McDiarmid* [1994, 1995] have observed FLR behavior up to latitudes corresponding to L =

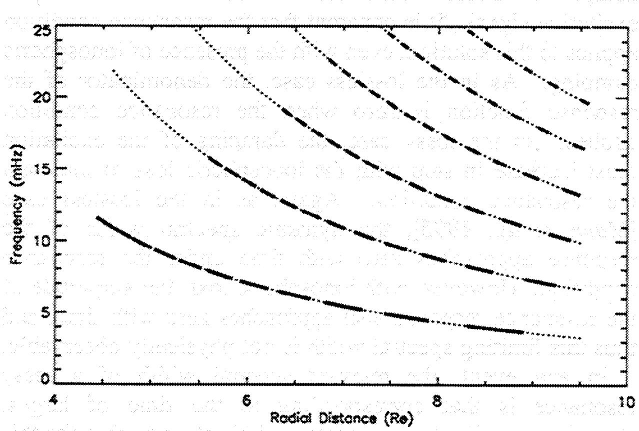

Figure 7. The same as Figure 5, except that $R = 0.900$ and α is constant for all resonances and set to the value corresponding to $\alpha/\omega_d = 0.05$ for the (j=1, k=1) resonance.

9. On this basis, $l_0 = 2$ was chosen. If l_0 is increased to 3, the outer (L) limit for the 2nd field-line harmonic (lowest curve) in the equivalent of Figure 5 is < 8.5. If the AMPTE/CCE plots do represent both odd and even field-line harmonics, as concluded for instance by *Engebretson et al.* [1986] and *Anderson et al.* [1989], the limit for $l_0 = 3$ is < 8.0, suggesting that l_0 is now too large. Nevertheless, the $l_0 = 3$ equivalent of Figure 5 shows gaps only slightly larger than those in Figure 5. Increasing α/ω_d to 0.07 produces a plot very similar to Figure 5 except for the shift of the right-hand side cutoffs to lower L values.

The net result is that it is possible in our model to find conditions under which, for any given field-line harmonic, the radial intervals of flux tubes excited in individual FLRs are almost all contiguous as a set (e.g., Figures 5 and 6). Many other possible conditions, however, do not produce a contiguous set (e.g., Figure 7). This is consistent with observation; not all AMPTE/CCE frequency versus L plots show continuous lines.

In this paper, V_A is a function only of x. In reality, it is also a function of z, and, in addition, the geomagnetic field is curved, i.e., quasi dipolar. In this circumstance, the structure of the cavity and waveguide modes becomes more complicated [see e.g., *Lee and Lysak*, 1989; *Smith et al.*, 1998]. The x and z dependency of these modes is not separable. One consequence is that each waveguide mode can potentially couple to more than one shear mode. This added coupling presents the possibility of filling in or diminishing whatever gaps remain in Figures 5 - 7. This, in turn, along with the noncritical cutoff condition for the solid lines in these figures, could relax the constraints on R and α/ω_d required to achieve a virtually gap-free version of Figures 5 - 7.

6. Discussion and Conclusions

In this paper, we have constructed a box model of the magnetosphere in which the shear mode frequency as a function of radial distance (L) is set to be consistent with the observations of *Poulter et al.* [1984]. The waveguide FLR frequencies of this model were determined as a function of the radial and field-line harmonic numbers using the theory of *Wright* [1994]. The equation for driven ionospherically damped field-line resonances was solved for a damped excitation signal. It is apparent that the resonance condition applies to this solution, even with the presence of ionospheric damping. As in the lossless case, the denominator of the response function is zero when the resonance condition applies. In the lossy case, the damping of the excitation must increase in step with the ionospheric loss to maintain the resonance condition. Again, as in the lossless case [*Mann et al.*, 1995], the dynamic spectral width of the response approaches zero with time under the resonance condition. However, with ionospheric loss, the amplitude of the resonance response also approaches zero with time, and thus this limiting spectral width is not physically observable.

In any event, the relevant spectral width of a lossy resonance is that corresponding to the time of largest resonance amplitude. A spectral width close to that for this time is most likely to be seen because detection of pulsations is weighted toward that time. As the ionospheric loss increases and/or as the decay rate of the excitation signal increases, the interval between the onset of the excitation and the time of maximum response decreases. Since the dynamic spectral width of the response decreases with the duration of excitation [*Mann et al.*, 1995], the effective spectral width of FLRs increases as ionospheric loss increases.

Finally, these results were synthesized to address the question of what characteristics might be revealed by FLRs excited by a quasi-random ensemble of sources forming an overall broadband excitation spanning roughly the frequency range 5-30 mHz. In particular, we wished to know what spectral properties might be observed. It was found possible under certain conditions that, for each of the even field-line harmonics 2-10, the spectral widths of FLRs corresponding to radial harmonics 1-12 were contiguous or overlapping at the 3 dB level. Assuming the visibility at AMPTE/ CCE of all the field-line harmonics would not change the results significantly.

If an AMPTE/CCE satellite were to pass through a magnetosphere excited consistent with the discussion above, it would see virtually all of the field lines ringing at multiple harmonic frequencies. These frequencies are not confined to the specific FLR values; the transient portion of the response on any given flux tube oscillates at the frequency of that flux tube. In other words, the transient portion of each of the set of FLR responses has a frequency which changes continuously with radial distance. The driven portion of the responses, however, are quantized very near one or other of the waveguide cutoff frequencies, but the combination will still yield a frequency which varies with radial distance [*McDiarmid and Allan*, 1990]. The exact form of the variation will depend on the time after excitation of the encounter of the satellite with the FLR and on the relative sizes of α and ω_i. However, if δ_r becomes more and more negative, $\tau_d = 1/\alpha$ becomes $<< \tau_i = 1/\omega_i$. Under this condition, the response rapidly becomes dominated by the transient component. This will accentuate the variation of frequency with radial distance as detected via the AMPTE/CCE analysis procedure [*Engebretson et al.*, 1986].

We have produced a possible explanation for the AMPTE/CCE observations in terms of waveguide modes. A propagating wave packet, which may be described as a combination of such modes, has now been observed [*Mann et al.*, 1998]. The propagation of such a wave packet across a given field line will provide a driver similar to that in equation (1). There may be a continual stream of similar wave packets, although probably not all of such large amplitude as the event studied by *Mann et al.* [1998]. The combination of these wave packets over time may be thought of as a quasi-random ensemble of short-duration excitation signals that our analysis shows may explain AMPTE/CCE satellite data. The signal data will have a resemblance to noise. Furthermore, the reason ground observations appear to differ from those made in space may have to do with the possibility that AMPTE/CCE type events correspond to $\delta_r <<$ 0 and have lower amplitude than the classic FLRs observed on the ground. If so, AMPTE/CCE type events would not stand out on the ground. Further work using simultaneous ground and satellite observations is clearly required to ascertain if the underlying mechanism is physically active.

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W. Allan, National Institute of Water and Atmospheric Research, P.O. Box 14-901, Kilbirnie, Wellington, New Zealand. (e-mail: w.allan@niwa.cri.nz)

D. R. McDiarmid, Herzberg Institute of Astrophysics, National Research Council, Ottawa, Ontario, Canada K1A0R6. (e-mail: mcdiarmid@canott.dan.sp-agency.ca)

A. N. Wright, Mathematical Institute, University of St Andrews, St Andrews, Fife KY16 9SS Scotland. (e-mail: andy@dcs.st_and.ac.uk)

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