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Excitation of resonant Alfvén waves in the magnetosphere by negative energy surface waves on the magnetopause

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Abstract. The instability of the magnetopause caused by the resonant interaction of a negative energy surface wave with Alfvén waves localized in the vicinity of the resonant magnetic surface is considered. The most important property of this instability is that it takes place for flow velocities in the magnetosheath below the Kelvin-Helmholtz (KH) threshold velocity, i.e., when there is no KH instability of the magnetopause. The magnetopause is modeled by an MHD tangential discontinuity with a magnetic-free plasma on the magnetosheath side and cold plasma on the magnetospheric side. It is shown that one of the two surface waves that propagate along the discontinuity when the shear velocity is smaller than the KH threshold velocity is a negative energy wave when the shear velocity is larger than a critical velocity. This negative energy wave propagates tailward in the magnetospheric frame, although it propagates sunward in the magnetosheath frame. When, in addition, a resonant condition is satisfied, the negative energy surface wave resonantly interacts with localized Alfvén waves. This interaction results in growth of both the surface wave and the Alfvén waves. The resonant condition can only be satisfied when the plasma density increases and, consequently, the Alfvén velocity decreases in the direction toward the magnetopause in its vicinity. The increment of the resonant instability is calculated under the assumption that the plasma density changes only in a slab in the vicinity of the magnetopause with the thickness much smaller than the wavelength. The possible observational signatures of the resonant instability are discussed.

1. Introduction

The fast-flowing magnetosheath provides the energy required for global magnetospheric convection and a range of ULF waves in the magnetosphere. Energy must be communicated across the magnetopause and a variety of mechanisms have been studied: *McKenzie* [1970] considered the transmission of MHD waves across the magnetopause; the displacement of the magnetopause due to pressure pulses [*Southwood and Kivelson*, 1990] or random magnetosheath buffeting has also been investigated [*Wright and Rickard*, 1995]; the magnetopause may also be unstable to a Kelvin-Helmholtz surface wave [*Dungey*, 1955; *Pu and Kivelson*, 1983, and references therein].

The Kelvin-Helmholtz instability on the flanks of the magnetopause (at dawn/dusk, and in the tail) has received much attention. These studies show that at dawn and dusk in the equatorial plane, where the magnetospheric field and sheath flow are approximately perpendicular, the boundary is particularly unstable. In the tail, where the flow is antisunward and the field generally parallel/antiparallel to the flow, the boundary is much more stable owing to magnetic tension. In this case there exists a critical velocity U_{KH} that must be exceeded before instability occurs. We present a new instability in this paper which can occur for sheath flow speeds below the critical velocity and is based upon the concept of negative energy waves.

The concept of negative energy waves turned out to be very fruitful when studying stability. In general, we talk about negative energy waves whenever we deal with a situation where waves become unstable and grow in presence of dissipation while they are neutrally stable in an ideal medium. An excellent review of the theory of negative energy waves in hydrodynamics is given by *Ostrovskii et al.* [1986]. Recently, *Ruderman and Goossens* [1995] studied negative energy waves on an MHD tangential discontinuity in an incompressible plasma. These authors showed that one of two sur-

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face waves propagating along the discontinuity when the shear velocity U is smaller than the Kelvin-Helmholtz (KH) threshold U_{KH} is a negative energy wave when $U_c < U < U_{\mathrm{KH}}$, where U_c is the critical velocity determined by the values of the equilibrium density and magnetic field at both sides of the discontinuity. When viscosity is present at one side of the discontinuity, the negative energy wave becomes unstable and grows.

In this paper we extend the analysis by *Ruderman and Goossens* [1995] to a more realistic situation. We consider a tangential MHD discontinuity with a magnetic-free plasma on one side and a cold magnetized plasma on the other. This configuration can be used as an approximate model in the vicinity of the magnetopause. We then use the interaction of a surface wave propagating along the discontinuity with Alfvén waves at the resonant position in the magnetosphere as a sink of energy causing instability of the negative energy wave. The details of the resonant fast/Alfvén wave coupling are similar to the calculations presented by *Tamao* [1965], *Southwood* [1974], and *Chen and Hasegawa* [1974], except that they considered standing waves, whereas our waves propagate along the magnetic field.

The paper is organized as follows. The next section gives a physical overview of the processes we study in the paper. Section 3 considers the KH instability of the discontinuity. In section 4 we consider negative energy surface waves. In section 5 the instability of the negative energy surface wave caused by resonant absorption in the magnetosphere is investigated. Section 6 discusses the main simplifying assumptions used in our study and the implications of our results, and section 7 contains our conclusions.

2. Physical Overview

In this section we give a physical overview of our calculations before providing mathematical details of our model in the remainder of the paper. The equilibrium we consider (see Figure 1) is a first approximation to the magnetotail, which is surrounded by a high-beta, antisunward flowing magnetosheath plasma and has low-beta tail lobes that contain magnetic field aligned with the sunward/antisunward directions. Thus the field and flow are parallel/antiparallel for the southern/northen tail lobes. In section 3 we discuss the Kelvin-Helmholtz instability of this configuration and find it to be stable for sheath slow speed U below a threshold value U_{KH} (approximately equal to the lobe Alfvén speed). When $U < U_{\mathrm{KH}}$, the stabilizing effect of magnetic tension prevents the instability from growing.

It is quite possible that there will be extended intervals when $U < U_{\mathrm{KH}}$ (U_{KH} is typically 600 km/s), and so it is natural to ask if there are other instabilities that may appear during these periods. The rest of the paper addresses this issue and begins by introducing the concept of negative energy waves. The energy density of a wave is dependent upon the frame of reference from

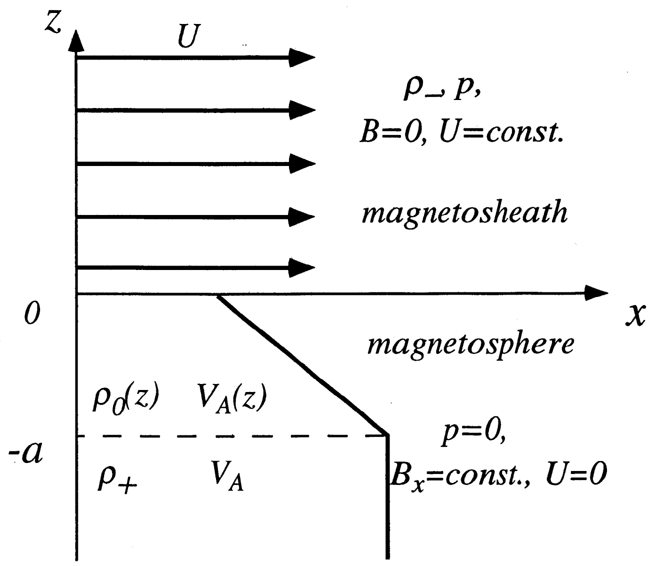

Figure 1. Sketch of the unperturbed state. The solid piecewise linear line in the lower part shows the dependence of the Alfvén velocity v_A on the distance from the heliopause, z. See text for variable descriptions.

which it is viewed, as is its frequency. We postpone the discussion of this dependence until section 5. In this section we use the reference frame where the magnetospheric plasma is at rest. In this reference frame the wave energy and frequency are determined by the velocity of the magnetosheath plasma. For example, when $U = 0$ the equilibrium in Figure 1 supports two surface waves propagating toward and away from Earth. If a small antisunward flow is included, the picture is modified slightly and the wave propagation is swept antisunward a little by the flow. Both waves still have a positive energy. Increasing the flow still further, we eventually reach a speed U_c at which the earthward propagating surface wave attains a negative energy density, when viewed from the magnetospheric frame. The flow speed U_c corresponds to the sheath flow speed which is sufficient to sweep back the earthward propagating wave, and for $U > U_c$ this mode actually propagates antisunward, as it is dominated by the sheath flow. The other surface mode continues to propagate away from Earth, but at even greater speeds than when $U = 0$. The critical speed U_c for negative energy waves to exist is of the order of the magnetosheath sound speed (approximately 100 km/s), so it is very easy to satisfy $U > U_c$.

A normal (positive energy) wave responds to the presence of dissipation by decaying in time. In contrast, a negative energy wave grows when there is dissipation or a sink of energy at rest relative to the observer. Thus inclusion of a magnetospheric sink of energy will damp the positive energy wave but cause the negative enrgy wave to become unstable. All this is posssible when $U_c < U < U_{\mathrm{KH}}$, so that there is no Kelvin-Helmholtz instability. When $U > U_{\mathrm{KH}}$, the negative energy wave becomes a zero-energy KH unstable wave. By "zero energy" we refer to the property that the energy density per unit area of the surface of discontinuity is identically zero.

In section 5 we consider a suitable sink of energy associated with the transition from tail lobe to magnetosheath. The surface wave may couple resonantly to Alfvén waves in this layer and will represent a sink of energy as far as the surface waves are considered. The possible observational signatures of the negative energy wave instability are discussed in section 7. These include the growth of unstable antisunward propagating magnetopause surface wave and the growth of the associated Alfvén resonance in the mantle (which also propagates antisunward).

3. The Study of KH instability

The KH instability of MHD tangential discontinuities has been intensively studied for a few decades. *Syrovatskii* [1957] and *Chandrasekhar* [1961] studied the stability of the MHD tangential discontinuity in ideal incompressible plasmas. They showed that there is a critical value of the jump in the equilibrium velocity, termed the KH threshold. The discontinuity is stable when the jump in the equilibrium velocity is below the KH threshold and unstable otherwise.

Fejer [1964] derived the dispersion equation determining the stability of the MHD tangential discontinuity in compressible plasmas and studied the particular case of slightly compressible plasma. Subsequently, many other particular cases were studied [e.g., *Gerwin*, 1968; *Duhau and Gratton*, 1973; *Gonzales and Gratton*, 1994a, b]. Application to the stability of the magnetopause was given, e.g., by *Sen* [1965], *Southwood* [1968], *McKenzie* [1970], *Kiyohumi and Saito* [1980], and *Pu and Kivelson* [1983].

We use the same equilibrium state as *McKenzie* [1970] (see Figure 1). The unperturbed MHD tangential discontinuity coincides with the xy plane in the Cartesian coordinates x, y, z. The half-space $z > 0$ is occupied by the magnetic-free plasma with the density ρ_+ and the pressure p. The half-space $z < 0$ is occupied by the cold plasma with the density ρ_-, permeated by a homogeneous magnetic field parallel to the x axis. The plasma in the upper half-space moves with the constant velocity U in the positive x direction, while the plasma in the lower half-space is at rest. The equilibrium quantities satisfy the condition of the total pressure balance at the discontinuity

$$p = \frac{B^2}{2\mu}, \tag{1}$$

where μ is the magnetic permeability of vacuum. For this particular type of equilibrium configuration the dispersion equation derived by *Fejer* [1964] is reduced to

$$\frac{\rho_+(\omega - k_x U)^2}{m_+} + \frac{\rho_-(\omega^2 - k_x^2 v_A^2)}{m_-} = 0 \tag{2}$$

with m_+ and m_- given by

$$m_+ = \left[k^2 - \frac{(\omega - k_x U)^2}{c_S^2}\right]^{1/2}, \quad m_- = \left(k^2 - \frac{\omega^2}{v_A^2}\right)^{1/2} \tag{3}$$

Here ω and $\mathbf{k} = (k_x, k_y, 0)$ are the (complex) frequency and the (real) wave vector of perturbations, respectively, which are taken to be proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega)$, with $\mathbf{r} = (x, y, z)$. The squares of the sound speed in the magnetic-free plasma and the Alfvén speed in the cold magnetic plasma are determined by $c_S^2 = \gamma p/\rho_+$ and $v_A^2 = B^2/\mu\rho_-$, with γ the adiabatic exponent. The surface wave decays as $z \to \pm\infty$, so the quantities m_+ and m_- satisfy the restriction

$$\Re(m_\pm) > 0, \tag{4}$$

where $\Re$ indicates the real part of a quantity. The tangential discontinuity is unstable when there is a root to (2) with a positive imaginary part.

Evidently, (3) are just the fast wave dispersion relations for the flowing field-free magnetosheath plasma and the steady cold magnetospheric plasma. Condition (4) implies that the fast waves are evanescent. Hence the surface waves are evanescent fast waves at both sides of the discontinuity.

Let us introduce the dimensionless quantities

$$\Omega = \frac{\omega}{k v_A}, \quad M = \frac{U}{v_A}, \quad \beta = \frac{c_S^2}{v_A^2}, \tag{5}$$

and the angle φ between the vector $\mathbf{k}$ and the x direction. Here M is the Alfvénic Mach number. In the new notation, (2) is rewritten as

$$\frac{\gamma(\Omega - M\cos\varphi)^2}{[\beta - (\Omega - M\cos\varphi)^2]^{1/2}} + \frac{2\beta^{1/2}(\Omega^2 - \cos^2\varphi)}{(1-\Omega^2)^{1/2}} = 0. \tag{6}$$

When deriving this equation we have used the formula $\rho_-/\rho_+ = 2\beta/\gamma$ that follows from (1). Squaring (6), we obtain

$$\gamma^2(\Omega^2 - 1)(\Omega - M\cos\varphi)^4 = 4\beta(\Omega^2 - \cos^2\varphi)^2[(\Omega - M\cos\varphi)^2 - \beta]. \tag{7}$$

This is a sixth-order polynomial equation with respect to Ω. It contains three parameters (γ can be considered as fixed), and it is a difficult problem to study the dependence of roots of this equation on the parameters. The additional complication is related to the possibility that some roots of (7) are spurious and do not satisfy the original equation (6). Instead, these spurious roots satisfy the equation obtained from (6) by substitution of a minus sign for the plus sign on the left-hand side.

McKenzie [1970] managed to study the dispersion equation in two particular cases only. In both cases the polynomial on the left-hand side of (7) is reduced to the product of a quadratic and a quartic. The first case corresponds to perturbations propagating along the equilibrium magnetic field ($\varphi = 0$), while the second case corresponds to perturbations propagating at the angle

$\varphi = \arcsin \frac{\gamma}{2}$. However, we cannot restrict the study of stability of the MHD tangential discontinuity to particular directions of propagation of disturbances. To calculate the KH threshold velocity U_{KH} we have to consider all possible disturbances.

To make analytical progress, we assume that the quantity ρ_-/ρ_+ is small or, equivalently, $\beta \ll 1$. This assumption is fairly well satisfied, at least for part of the tail's magnetopause. Note that a similar assumption was made by *Ruderman and Fahr* [1993, 1995] when studying the stability of the heliopause, which is the tangential discontinuity separating the solar wind compressed at the inner shock from the interstellar plasma compressed at the outer shock. The viable assumption that the density of the solar wind is much smaller than the density of the interstellar plasma enabled these authors to carry out a complete analytical study of the heliopause stability. The analysis in this section is quite similar to that by *Ruderman and Fahr* [1993, 1995].

Since $\beta \ll 1$, the roots of (7) are close to those for $\beta = 0$, i.e., either to ± 1 or to $M \cos\varphi$, which is the multiple root. The two roots close to ± 1 are obviously real and not interesting for studying stability. Let us consider the four roots close to $M \cos\varphi$. They can be looked for in the form $\Omega = M\cos\varphi + \delta\Omega$, where $|\delta\Omega| \ll 1$. Substitution of this expression into (7) yields the leading order approximation

$$\gamma^2(M^2\cos^2\varphi - 1)(\delta\Omega)^4 - 4\beta(M^2-1)^2(\delta\Omega)^2\cos^4\varphi + 4\beta^2(M^2-1)^2\cos^4\varphi = 0. \quad (8)$$

It is straightforward to see that this equation has two real and two purely imaginary complex conjugated roots when $M^2\cos^2\varphi < 1$, and all of these four roots are of the order $\beta^{1/2}$. Hence (7) has four real roots and two complex conjugated roots of the form

$$\Omega = M\cos\varphi \pm i\kappa\beta^{1/2} \quad (9)$$

with $\kappa > 0$, $\kappa \sim 1$. As mentioned previously, any root of (7) satisfies either (6) or the equation obtained from (6) by substituting a minus sign for the plus sign. The direct substitution of (9) into (6) shows that the real parts of the two terms on the left-hand side of (6) have the same signs when $M < 1$, while they have the opposite signs when $M > 1$. This implies that roots given by (9) satisfy (6) when $1 < M < 1/\cos\varphi$, while they are spurious when $M < 1$. Hence (6) has only real roots when $M < 1$, while it has a complex root with positive imaginary part when $1 < M < 1/\cos\varphi$. This analysis leads us to the conclusion that the discontinuity is stable when $M < 1$ and unstable when $M > 1$, so that we obtain for the Kelvin-Helmholtz threshold $M_{\mathrm{KH}} = 1$. Note that this result coincides with that obtained by *McKenzie* [1970] for perturbations propagating parallel to the background magnetic field. However, this result is approximate since we used β as a small parameter. Therefore we only conclude that M_{KH} is close to 1. This conclusion is sufficient for what follows.

4. Negative Energy Surface Waves

The concept of negative energy waves was used in plasma physics long ago [see, e.g., *Briggs*, 1964; *Bekefi*, 1966; *Coppi et al.*, 1969; *Davidson*, 1972]. Later, this consept was applied to hydrodynamic waves, in particular, to waves on the surface of two fluids [see, e.g., *Cairns*, 1979; *Craik*, 1985; *Ostrovskii et al.*, 1986]. *Ryutova* [1988] developed a theory of negative energy waves in magnetically structured plasmas, and *Joarder et al.* [1997] applied this theory to waves in the solar atmosphere. *Ruderman and Goossens* [1995] studied instability of a tangential MHD discontinuity in an incompressible plasma related to the presence of a negative energy surface wave. In magnetospheric physics, *McKenzie* [1970] used the concept of negative energy waves when studying overreflection of waves coming from the magnetosheath from the magnetopause. However, to the best of our knowledge, this concept was not used when studying the magnetopause stability. Since the concept of negative energy waves is relatively new for the magnetospheric community, we give a short introduction to the theory of negative energy waves, with particular emphasis on surface waves on magnetic interfaces.

A wave has negative energy if its growth from an unperturbed state requires that energy be extracted from the system rather than fed into it; that is, exciting the wave decreases the system energy. Let us derive the expression for the energy of a surface wave on a stable tangential MHD discontinuity. In what follows we exploit the modified procedure used by *Cairns* [1979] for deriving the energy of a wave on an interface between two incompressible fluids. As before we consider a plane MHD tangential discontinuity with a cold magnetic plasma at one side and a magnetic-free plasma at the other side. The linear MHD equations for a cold plasma can be reduced to the set of two equations for the z component of the velocity u and the perturbation of the magnetic pressure $P' = B_0 B_x'/\mu$

$$\frac{du}{dz} = \frac{i\omega(\omega^2 - v_{A0}^2 k^2)}{\rho_0 v_{A0}^2(\omega^2 - \omega_A^2)} P', \quad (10)$$

$$\frac{dP'}{dz} = \frac{i\rho_0(\omega^2 - \omega_A^2)}{\omega} u. \quad (11)$$

Here perturbations of all quantities are taken to be proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega)$, and the prime indicates the perturbation of a quantity. The square of the Alfvén velocity is given by $v_{A0}^2(z) = B^2/\mu\rho_0(z)$, and the Alfvén frequency is given by $\omega_A = v_{A0}(z)k_x = v_{A0}(z)k\cos\varphi$, where $\rho_0(z)$ is the unperturbed density. In this section, $\rho_0(z) = \rho_-$ and $v_{A0} = v_A$, although (10) and (11) for the description of plasma motions in an inhomogeneous plasma have taken the dependence of the unperturbed density on z into account when deriving them.

The motions of the magnetic-free plasma occupying the region $z > 0$ is described by

$$\frac{du}{dz} = i\frac{(\omega - Uk\cos\varphi)^2 - c_S^2k^2}{\rho_+ c_S^2(\omega - Uk\cos\varphi)}P', \qquad (12)$$

$$\frac{dP'}{dz} = i\rho_+(\omega - Uk\cos\varphi)u, \qquad (13)$$

where now P' is the perturbation of the plasma pressure.

Consider now a perturbation displacement of the tangential discontinuity in the form

$$z = \tilde{\eta}(t, x, y) = \Re[\eta\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)], \qquad (14)$$

where η is a real quantity. Perturbations of all quantities are taken in the form $\tilde{f} = \Re[f(z)\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)]$. It is straightforward to obtain

$$P'_- = D_-(\omega, \mathbf{k})\eta \equiv \frac{\rho_-(\omega^2 - \omega_A^2)}{m_-}\eta, \qquad (15)$$

$$P'_+ = D_+(\omega, \mathbf{k})\eta \equiv -\frac{\rho_+(\omega - Uk\cos\varphi)^2}{m_+}\eta, \qquad (16)$$

where $P'_\pm = P'(z = \pm 0)$. When deriving these equations, we have used the kinematic boundary conditions

$$u_- = -i\omega\eta, \quad u_+ = i(kU\cos\varphi - \omega)\eta. \qquad (17)$$

Then the condition of total pressure balance results in the dispersion equation

$$D(\omega, \mathbf{k}) \equiv D_+(\omega, \mathbf{k}) - D_-(\omega, \mathbf{k}) = 0, \qquad (18)$$

which coincides with (2).

Now suppose that for a wavenumber $\mathbf{k}_0$ there exists a real solution ω_0 to (18). Let us impose a surface force to the surface of the discontinuity that makes the initial perturbation in form (14) exponentially grow, so that the wave amplitude is $\eta e^{\omega' t}$ with real positive ω'. This surface force can be maintained by, e.g., exciting an additional surface current on the surface of the discontinuity. We assume that initially the amplitude of the perturbation η is so small that the energy of the perturbation can be neglected. Now $\omega = \omega_0 + i\omega'$, and, for the sake of simplicity, we assume that the surface force is small, so that $\omega' \ll |\omega_0|$. Equations (15) and (16) are still valid, however, owing to the presence of the surface force $P'_- \neq P'_+$ and $D(\omega, \mathbf{k}_0) \neq 0$, so that the exponentially growing perturbation is not an eigenmode.

Let us now consider the process where the external force is being imposed for a finite time interval T and calculate the energy of a perturbation created by this force. This perturbation is once again a normal mode with the wavenumber $\mathbf{k}_0$ and frequency ω_0, however, with the amplitude $\eta_T = \eta e^{\omega' T}$. To calculate the energy of the perturbation, we calculate the work done on the plasma by the external force. The unit vector of the normal to the surface of the discontinuity is $\mathbf{n} = -\nabla\tilde{\eta} + \mathbf{z}_0$, where $\mathbf{z}_0$ is the unit vector in the z direction and we have neglected nonlinear terms. The projection of the velocity on the normal is $\tilde{u}_+ - U\partial\tilde{\eta}/\partial x = \partial\tilde{\eta}/\partial t$. The rate at which the work is done on the upper plasma per unit area in the x, y plane is $\tilde{P}'_-\partial\tilde{\eta}/\partial t$. The similar calculation gives for the work done on the lower plasma, $-\tilde{P}'_+\partial\tilde{\eta}/\partial t$. Then the rate at which the total work is done by the external force on the plasma per unit area in the x, y plane is

$$\frac{\partial\tilde{\eta}}{\partial t}(\tilde{P}'_- - \tilde{P}'_+). \qquad (19)$$

We use (14)–(16), (18), and the approximation

$$D_\pm(\omega, \mathbf{k}_0) \approx D_\pm(\omega_0, \mathbf{k}_0) + i\omega'\frac{\partial D_\pm}{\partial\omega_0} \qquad (20)$$

to obtain

$$\frac{\partial\tilde{\eta}}{\partial t}(\tilde{P}'_- - \tilde{P}'_+) = \omega_0\eta^2\frac{\partial D}{\partial\omega_0}\omega' e^{2\omega' t}\sin^2(\mathbf{k}\cdot\mathbf{r} - \omega_0). \quad (21)$$

When deriving this expression, we have neglected ω' in comparison with ω_0. The energy of the perturbation averaged over the wavelength in the x and y direction is W,

$$\frac{dW}{dt} = \tfrac{1}{2}\omega_0\omega' e^{2\omega' t}\frac{\partial D}{\partial\omega_0}\eta^2. \qquad (22)$$

Integrating this equation and taking into account that $W(0) \approx 0$, we obtain

$$W(T) = \tfrac{1}{4}\omega_0\frac{\partial D}{\partial\omega_0}\eta_T^2. \qquad (23)$$

Hence the wave is a negative energy wave if

$$\omega_0\frac{\partial D}{\partial\omega_0} < 0, \qquad (24)$$

and a positive energy wave otherwise.

Let us now assume that dissipation is present. Then the wave amplitude and energy are changing and $W(t)$ is given by (23) with $\eta(t)$ substituted for η_T. Owing to the presence of dissipation, the wave energy is decreasing, $dW/dt < 0$. For a positive energy wave it results in the wave damping, $d\eta/dt < 0$. However, for a negative energy wave the decrease of the wave energy means that its absolute value grows. As a result, the wave amplitude grows, $d\eta/dt > 0$. Note also that negative energy waves possess the peculiar property that their energy flux is antiparallel to the group velocity.

Now we apply the obtained results to surface waves on the magnetopause. To ensure that the discontinuity is stable, we assume that $M < M_{\mathrm{KH}}$, where M_{KH} is close to 1. Then, in accordance with the results of section 3, (7) has two complex conjugated roots and four real roots. The complex roots are spurious in the sense that they do not satisfy the original dispersion equation (6). The real roots are given by

$$\Omega_1 = 1 + \mathcal{O}(\beta), \quad \Omega_2 = -1 + \mathcal{O}(\beta), \qquad (25)$$

$$\Omega_\pm = (M \pm \beta^{1/2}\xi)\cos\varphi + \mathcal{O}(\beta), \qquad (26)$$

where

$$\xi^2 = \frac{2(1-M^2)}{\gamma^2(1-M^2\cos^2\varphi)}\{(M^2-1)\cos^2\varphi + [(1-M^2)^2\cos^4\varphi + \gamma^2(1-M^2\cos^2\varphi)]^{1/2}\}. \quad (27)$$

It is straightforward to show that the roots given by (25) are spurious, while those given by (26) are not spurious and satisfy dispersion equation (6). Note that $\xi^2\cos^2\varphi < 1$. It follows from this inequality and the condition $M < 1$ that m_+ and m_- are real, so that the roots given by (26) correspond to two surface waves propagating along the tangential discontinuity in the opposite directions in the coordinate system moving together with the upper (magnetosheath) plasma.

We see that (18) has exactly two real roots, $\omega_\pm = kv_A\Omega_\pm$, for any fixed $\mathbf{k}$. The function $D(\omega, \mathbf{k})$ is real when

$$k_xU - kc_S < \omega < k_xU + kc_S. \quad (28)$$

We have used the condition $M < M_{\rm KH} < 1$ when deriving these inequalities. Since the roots $\omega_\pm$ correspond to surface waves with real $m_\pm$, they are in the interval, determined by inequalities (28). It is straightforward to show that $D(\omega,\mathbf{k}) \to \infty$ when $\omega \to k_xU \pm kc_S$. Hence $D(\omega,\mathbf{k})$ is positive for $\omega < \omega_-$ and for $\omega > \omega_+$, while it is negative for $\omega_- < \omega < \omega_+$. These facts imply that

$$\left.\frac{\partial D}{\partial \omega}\right|_{\omega=\omega_-} < 0, \quad \left.\frac{\partial D}{\partial \omega}\right|_{\omega=\omega_+} > 0. \quad (29)$$

Since $\omega_+ > 0$, it follows from (24) and (29) that the surface wave with the frequency ω_+ is always a positive energy wave. The surface wave with the frequency ω_- is a positive energy wave when $\omega_- < 0$ and a negative energy wave when $\omega_- > 0$. Hence the wave propagating backward with respect to the upper plasma is a negative energy wave when it propagates forward with respect to the lower plasma. Using (26) and (27), it is straightforward to show that $\omega_- > 0$ when $M > M_c$, where the critical Mach number is given by

$$M_c = \frac{1}{\gamma}\{2\beta[(\gamma^2+\cos^4\varphi)^{1/2} - \cos^2\varphi]\}^{1/2} + O(\beta). \quad (30)$$

We can use another approach to calculate M_c. We simply substitute $\Omega = 0$ into dispersion equation (7) and immediately obtain the equation determining M_c. From this equation we find that (30) for M_c is actually exact and does not contain a correction $O(\beta)$.

Summarizing the present discussion, we state that the surface wave propagating backward with respect to the magnetosheath plasma is either a positive or negative energy wave depending on whether the Mach number is smaller or larger than M_c or, in other words, whether the shear velocity U is smaller or larger than the critical velocity $U_c = M_c v_A$. It follows from (30) that $M_c = O(\beta^{1/2})$, so that U_c is of the order c_S. Hence for almost all values of the shear velocity U, except values from the small range from 0 to a value of the order c_S, there is a negative energy wave propagating on the discontinuity.

The wave energy depends on the choice of a moving coordinate system, so that a wave can be a negative energy wave in one coordinate system and a positive energy wave in another. To illustrate this statement, let us consider the same surface waves, however, now in a coordinate system where the magnetic-free plasma is at rest and the magnetic plasma moves in the x direction with the velocity $-U$. Owing to the Doppler shift, the frequency of the background wave in this coordinate system is $\omega_- - Uk_x = -kv_A\xi\cos\varphi < 0$, so that this wave is always a positive energy wave. However, the wave stability is, of course, independent of the coordinate system. A very throrough discussion of this problem is given, e.g., by *Ostrovskii et al.* [1986], so here we do not embark on a long discussion of it. We only note that, in accordance with *Ostrovskii et al.* [1986], in the case where dissipation takes place at one side of the discontinuity only, a surface wave is unstable if it is a negative energy wave in the coordinate system where the dissipative plasma is immovable.

The best way to explain the physical meaning of U_c is to consider the situation where the magnetospheric plasma is dissipative while the magnetosheath plasma is ideal. In this case the surface wave propagating backward with respect to the magnetosheath plasma is stable when $U < U_c$ and unstable otherwise (recall that we consider $U < U_{\rm KH}$, so that the surface wave is neutrally stable in an ideal plasma). When $U > U_c$, the surface wave extracts energy from the shear flow. The interaction between the shear flow and the surface wave is provided by dissipation in the magnetospheric plasma.

5. Resonant Instability of Surface Waves

Let us now consider the equilibrium state where the plasma density is not constant in the magnetosphere but varies in the layer $-a < z < 0$. The density is $\rho_0(z)$ in the interval $[-a, 0]$, and it is ρ_- for $z < -a$. The density is continuous for $z < 0$, so that $\rho_0(-a) = \rho_-$. In what follows we only consider surface waves with a wavelength much larger than the thickness of the inhomogeneous layer, $ak \ll 1$. Then the analysis in the previous sections 3 and 4 is approximately valid. The variation in the density in the inhomogeneous layer leads to the variation in the Alfvén velocity. As a result, the Alfvén resonant condition can be satisfied in the inhomogeneous layer for some values of the phase velocity of the surface wave. When this condition is satisfied, resonant absorption of the wave energy takes place. Resonant absorption provides the energy sink in the magnetosphere and leads to growth of the amplitude of the negative energy wave. In this section we study this process in detail.

In what follows we assume that $\rho_0(z)$ is monotonic. Then the Alfvén velocity $v_{A0}(z) = B[\mu\rho_0(z)]^{-1/2}$ in the inhomogeneous layer varies from $v_A = B(\mu\rho_-)^{-1/2}$

at $z = -a$ to $v_{A1} = B[\mu\rho_0(0)]^{-1/2}$ at $z = 0$. A surface wave is in resonance with the local Alfvén oscillations at a position $z = z_A$ if its frequency coincides with the local Alfvén frequency, $\omega_A(z_A) = v_{A0}(z_A)k\cos\varphi$. Hence the resonant condition takes the form $\Omega = V(z_A)\cos\varphi$, where $V(z) = v_{A0}(z)/v_A$. We are only interested in Alfvén resonance for negative energy waves, so that we take $\Omega = \Omega_-$ and assume that $M_c < M < M_{\rm KH}$. Then with the use of (26) we rewrite the resonant condition as

$$V(z_A) = M - \beta^{1/2}\xi. \qquad (31)$$

As shown in section 2, $M_{\rm KH}$ is close to unity, so that we have to take $M < 1$. Then it follows from (31) that the Alfvén resonant condition can be satisfied only if $v_{A0}(z)$ is monotonically decreasing and, consequently, $\rho_0(z)$ is monotonically increasing. In addition, the condition $v_{A1} < v_A(M - \beta^{1/2}\xi)$ has to be satisfied.

Here we have to make one note. The magnetospheric community is more used to considering resonant oscillations of closed magnetic field lines in the magnetosphere. In this case every magnetic line has its own eigenfrequency. Its oscillations are resonantly excited by a periodic external or internal energy source if the frequency spectrum of this source contains a frequency that matches a natural frequency of the magnetic field line. However, resonant oscillations can be also excited in an unbounded plasma [see, e.g., *Goossens*, 1991; *Allan and Wright*, 1998]. In this case there are no eigenfrequencies of the magnetic field lines and it is more convenient to discuss the Alfvén resonance in terms of the phase velocity. To be more specific, we consider the excitation of the Alfvén resonance by a surface wave propagating on the magnetopause. In this case the Alfvén resonance takes place at a magnetic field line where the phase velocity of the surface wave matches the local Alfvén velocity. Varying the wavelength of the surface wave, we can obtain resonant oscillations with any frequency. Hence the only difference between Alfvén resonances in bounded and unbounded plasmas is that in bounded plasmas, resonant frequencies of a specific magnetic line are quantized while in unbounded plasmas they are continuous.

Let us now calculate the instability increment of the negative energy wave under the assumption that the resonant condition (31) is satisfied. Linear ideal MHD predicts that the wave amplitude is infinite at the resonant position, $z = z_A$. Dissipation removes the singularity from the linear MHD equations. However, when dissipation is weak, it is only important in a thin dissipative layer embracing the ideal resonant position. The wave motion in the dissipative layer is still characterized by very large amplitudes. Outside the dissipative layer, the wave motion can be described by the linear ideal MHD equations. This observation led *Sakurai et al.* [1991] to devise a method for obtaining the solutions describing resonant waves that does not require solving the full set of dissipative MHD equations in the whole volume occupied by a plasma [see also *Goossens et al.*, 1995; *Goossens and Ruderman*, 1995]. In accordance with this method the ideal MHD equations on each side of the dissipative layer and the dissipative MHD equations in the dissipative layer are solved separately. Then the dissipative layer is considered as a surface of discontinuity, and the dissipative solution is used to obtain connection formulas, which are expressions for the jumps across the dissipative layer in the normal components of the velocity and in the perturbation of the total pressure. These connection formulas are then used to connect the ideal solutions on each side of the dissipative layer.

Sakurai et al. [1991] and *Goossens et al.* [1995] obtained connection formulas for Alfvén dissipative layers in one-dimensional axisymmetric equilibria. Their approach was to solve the dissipative equations across the resonance layer and determine the asymptotic changes in wave quantities across this layer for small dissipative coefficients. It is straightforward to rewrite their connection formulas for one-dimensional planar equilibria and obtain

$$[P'] = 0, \quad [u] = -\frac{\pi\omega k^2 P' \sin^2\varphi}{\rho_A|\Delta|}, \qquad (32)$$

where $\Delta = d\omega_A^2/dz$ calculated at $z = z_A$, $\rho_A = \rho_0(z_A)$, and the brackets indicate the jump in a quantity across the dissipative layer. Note that the right-hand side of the second equation (32) is of the order ak. Traditionally, the magnetospheric community has used the Landau-damping recipe for calculating the jump in the solution; for the present equations this requires adding a small imaginary part to the frequency, causing the solutions to grow in time. Equations (10) and (11) may then be integrated directly along the real z axis past the singularity, and the limit of vanishing imaginary frequency can be taken. This is described in more detail elsewhere, e.g., *Thompson and Wright* [1993, section 4], and we note that this procedure and their equation (41) reproduces the jump conditions above.

Connection formulas similar to (32) were obtained by *Sakurai et al.* [1991] and *Goossens et al.* [1995] for plasmas where the only dissipative process is resistivity. However, it is not difficult to repeat their derivation for plasmas with other dissipative processes, e.g., with dissipation provided by Pedersen conductivity in the ionosphere [*Wright and Allan*, 1996]. It is a general principle that the connection formulas in linear MHD do not depend on particular types of dissipative processes operating in the dissipative layer.

The motions of the magnetic plasma are described by (10) and (11). We eliminate u from these equations to arrive at

$$\frac{d}{dz}\frac{1}{\rho_0(\omega^2 - \omega_A^2)}\frac{dP'}{dz} + \frac{\omega^2 - v_{A0}^2k^2}{\rho_0 v_{A0}^2(\omega^2 - \omega_A^2)}P' = 0. \qquad (33)$$

The ratio of the second term in (33) to the first one is of the order $(ak)^2$. Since we consider the long-wavelength approximation and assume $ak \ll 1$, we can neglect the second term and obtain

$$P' = P'_0 + A\int_0^z \rho_0(\bar{z})[\omega^2 - \omega_A^2(\bar{z})]\,d\bar{z}, \qquad (34)$$

where P'_0 and A are constant. Equations (10) and (11) are only valid outside the dissipative layer, so that, in general, P'_0 and A take different values in the regions $-a < z < z_A$ and $z_A < z < 0$. However, the first connection formula (32) ensures that A and P'_0 take the same values in the whole region, $-a < z < 0$.

We use (10) to get in the region $-a < z < 0$

$$u = \begin{cases} u(-a) + \displaystyle\int_{-a}^{z} \frac{i\omega[\omega^2 - k^2 v_{A0}^2(\bar{z})]P'(\bar{z})\,d\bar{z}}{\rho_0(\bar{z})v_{A0}^2(\bar{z})[\omega^2 - \omega_A^2(\bar{z})]}, & z < z_A, \\ u(0) - \displaystyle\int_{z}^{0} \frac{i\omega[\omega^2 - k^2 v_{A0}^2(\bar{z})]P'(\bar{z})\,d\bar{z}}{\rho_0(\bar{z})v_{A0}^2(\bar{z})[\omega^2 - \omega_A^2(\bar{z})]}, & z > z_A. \end{cases} \qquad (35)$$

Then it is straightforward to obtain

$$[u] = u(0) - u(-a) - i\omega P'_0 \mathcal{P}\int_{-a}^{0} \frac{(\omega^2 - k^2 v_{A0}^2)\,dz}{\rho_0 v_{A0}^2(\omega^2 - \omega_A^2)} + O(k^2a^2), \qquad (36)$$

where $\mathcal{P}$ indicates the Cauchy principal part of an integral. When deriving this equation, we have used the estimate that the second term on the right-hand side of (34) is of the order ak. Comparison of the second equation (32) and (36) yields

$$u(0) - u(-a) = i\omega P'_0 \mathcal{P}\int_{-a}^{0} \frac{(\omega^2 - k^2 v_{A0}^2)\,dz}{\rho_0 v_{A0}^2(\omega^2 - \omega_A^2)} - \frac{\pi\omega k^2 P'_0 \sin^2\varphi}{\rho_A|\Delta|} + O(k^2a^2). \qquad (37)$$

To derive the dispersion equation determining the dependence of ω on k, we need to express $u(0)$ and $u(-a)$ in terms of P'_0. The plasma motion in the magnetosphere outside the inhomogeneous layer ($z < -a$) is determined by (10) and (11) with ρ_-, v_A, and $v_A k\cos\varphi$ substituted for ρ_0, v_{A0}, and ω_A. It is straightforward to obtain the solution to this set of equations vanishing as $z \to -\infty$. We then use this solution and the continuity of u and P' at $z = -a$ to obtain

$$u(-a) = -\frac{i\omega m_- P'_0}{\rho_-(\omega^2 - v_A^2 k^2\cos^2\varphi)}. \qquad (38)$$

The motion of the magnetic-free plasma is described by the set of linear gasdynamic equations (12) and (13). Once again, it is straightforward to obtain the solution to this set of equations now vanishing as $z \to \infty$. With the use of this solution, the continuity of P' at $z = 0$, and the boundary conditions (17), we get

$$u(0) = \frac{i\omega m_+ P'_0}{\rho_+(\omega - Uk\cos\varphi)^2}. \qquad (39)$$

Substitution of (38) and (39) into (37) yields

$$\frac{m_+}{\rho_+(\omega - Uk\cos\varphi)^2} + \frac{m_-}{\rho_-(\omega^2 - v_A^2k^2\cos^2\varphi)} = \mathcal{P}\int_{-a}^{0} \frac{(\omega^2 - k^2v_{A0}^2)\,dz}{\rho_0 v_{A0}^2(\omega^2 - \omega_A^2)} + \frac{\pi i k^2\sin^2\varphi}{\rho_A|\Delta|} + O(k^2a^2). \qquad (40)$$

The ratio of the right-hand side of (40) to the left-hand side is of the order $ak \ll 1$. This fact enables us to use the regular perturbation method to solve this equation. In accordance with this method we look for the solution in the form $\omega = \bar{\omega} + \omega'$, where $|\omega'| \ll |\bar{\omega}|$. In the first-order approximation we take $\omega = \bar{\omega}$ and neglect the right-hand side of (40). As a result, we arrive at dispersion equation (2) for surface waves on an MHD discontinuity. We take the solution $\omega = \Omega_- v_A k$ to this equation. In the next approximation we calculate the correction ω' to order $\beta^{1/2}(ak)$

$$\Im(\omega') = \beta^{1/2}\frac{\pi\rho_- k^4 v_A^3\gamma^2\xi^5\sin^2\varphi\cos^3\varphi}{4\rho_A|\Delta|(2 - \xi^2\cos^2\varphi)} \times \frac{(1 - M^2\cos^2\varphi)^{1/2}}{1 - M^2}. \qquad (41)$$

We see that $\Im(\omega') > 0$, so that, in complete agreement with the general theory, the negative energy wave becomes unstable in presence of the energy sink owing to resonant absorption. Let us recall that (41) was derived under the assumption that the resonant condition (31) is satisfied, i.e., that there is resonance between the negative energy surface wave and the local Alfvén oscillations at a position $z = z_A$. The $\Im(\omega') = 0$ when either $\varphi = 0$ or $\varphi = \pi/2$, so that only obliquely propagating waves are unstable.

Note that similar problems were studied by *Hollweg et al.* [1990], *Yang and Hollweg* [1991], and *Tirry et al.* [1998]. *Hollweg et al.* [1990] investigated resonant instability of surface waves in the presence of a shear flow in incompressible plasmas, and *Yang and Hollweg* [1991] explored it in cold plasmas. These authors did not use the concept of negative energy waves; however, it is straightforward to show that the instability found by them is related to the presence of negative energy waves. *Tirry et al.* [1998] studied the resonant instability of the tangential MHD discontinuity numerically. The main difference of their paper from ours is that they had constant mangetic field and finite beta plasmas at both sides of the discontinuity.

Harrold et al. [1990] studied resonant absorption of surface waves propagating on the magnetopause in the plasma sheet boundary layer of the magnetotail. The main difference of the study by *Harrold et al.* [1990] from ours is that they did not investigate the magnetopause instability caused by the resonant absorption. Instead, they assumed that there is a marginally stable surface wave propagating along the magnetopause and studied the heating of the plasma sheet boundary layer due to resonant absorption.

The instability considered in this section is a typical example of instability related to the resonant interaction of the negative and positive energy waves. As we have seen, the backward wave is a negative energy wave in the coordinate system where the magnetic plasma is at rest. The dispersion equation for the localized Alfvén waves is

$$D_A(\omega, \mathbf{k}) \equiv \omega^2 - \Omega_-^2 v_A^2 k^2 = 0,$$

so that, in accordance with (23), they are positive energy waves. If we once again consider the new moving coordinate system where the magnetic-free plasma is at rest, then the backward surface wave is a positive energy wave, while the Alfvén waves are negative energy waves. The result of their interaction is, of course, the same as previously, namely, the growth of both positive and negative energy waves.

6. Discussion

In this paper we use a very crude model of the magnetopause. Therefore we need to discuss how our results would be changed if we use more realistic model.

1. We assume that the magnetosheath plasma is magnetic free. In reality, there is usually a magnetic field on the magnetosheath side. *Tirry et al.* [1998] numerically studied the other extreme case where there is the same constant magnetic field on both sides of the discontinuity. These authors obtained results that qualitatively coincide with ours. Comparison of our results with results obtained by *Tirry et al.* [1998] indicates that inclusion of the magnetic field in the magnetosheath can only quantitatively change our results, i.e., the exact expressions for U_c and for the instability increment. However, it does not change our results qualitatively.

2. We used the assumption that the magnetospheric plasma is cold, which is not satisfied, at least in the mantle. Once again, we refer to *Tirry et al.* [1998], who considered the finite beta plasma at the both sides of the discontinuity. Comparison of their results with ours shows that the account of the finite temperature of the magnetospheric plasma does not qualitatively change our results.

3. We model the magnetopause by an infinitely thin MHD discontinuity, which is definitely an idealization. Then the question arises: how will our results change if we take the finite thickness of the magnetopause into account? To answer this question, we refer to *Hollweg et al.* [1990] and *Yang and Hollweg* [1991]. These authors considered the resonant instability of MHD shear flows with the velocity varying in a finite layer with thickness d. They then used the long-wavelength approximation; that is, they assumed that $kd \ll 1$. As a result, they obtained the expression for the critical velocity U_c independent of d and the instability increment proportional to kd. *Hollweg et al.* [1990] and *Yang and Hollweg* [1991] assumed that the characteristic scales of variation of the shear velocity and the Alfvén speed are the same and equal d. In a more general case they are different, the first one being d and the second one being a. The case considered in our paper corresponds to $a \gg d$. Since a is the thickness of the mantle and d is the thickness of the magnetopause, this estimate seems to be reasonable. If we take the finite thickness of the magnetopause into account, we obtain, in the long-wavelength approximation ($ka \ll 1$), corrections of the order d/a to the instability increment and of the order $kd \ll d/a$ to U_c. Obviously, these corrections are not important.

4. The other important question that is related to modeling the magnetopause by a current/vortex sheet is, what is the wavelength of the most unstable perturbation? It is well known that in the case where an MHD discontinuity separates two homogeneous plasmas the criterion of the KH instability is independent of the perturbation wavelength. The instability increment is proportional to the wavenumber, so that the shorter a perturbation is, the faster it grows. This unphysical behavior of the unstable perturbations, especially the property of the instability increment to tend to infinity when the wavelength is increased, inspired *Lerche* [1966] to claim that modeling the magnetopause by an MHD tangential discontinuity is inadequate.

Walker [1981] addressed the effect of a finite width velocity shear on magnetopause stability. He found that the most unstable perturbations are those with wavelengths of the order of the transitional layer thickness. Perturbations with wavelengths much smaller than the transitional layer thickness are stable. However, it is interesting to note that for perturbations with wavelengths much larger than the transitional layer thickness the growth rate is the same as that in the case of MHD discontinuity. *Walker* [1981] assumed that the regions on either side of the vortex layer were homogeneous.

In the case where the two plasmas are once again separated by an MHD tangential discontinuity, however, at least one of these plasmas is inhomogeneous in the direction perpendicular to the discontinuity, so the situation is essentially the same as in the case where the both plasmas are homogeneous. The reason is that the amplitudes of unstable surface waves exponentially decay with distance from the discontinuity. The characteristic scale of this decay is the surface wave wavelength. As a result, the inhomogeneity does not affect perturbations with wavelengths much smaller than the characteristic scale of the inhomogeneity. The stability of an MHD tangential discontinuity with an inhomogeneous plasma at one side of it was studied by *Fujita et al.* [1996]. These authors studied the stability of the magnetopause, modeling it by an MHD tangential discontinuity and assuming that the magnetosheath plasma is homogeneous while the magnetospheric plasma is inhomogeneous, with the Alfvén speed growing with the distance from the magnetopause. Their numerical results show that for not very large values of the Alfvèn Mach number of the magnetosheath flow the growth rate of perturbations increases when the wavenumber k is increased. This result is in agreement with the theoretical prediction that for large k the growth rate is proportional to k. However, for large values of the Alfvén Mach number the growth rate first increases when k is increased, reaches its maximum value, and then slowly decreases. The difference arises from the fact that *Fujita et al.* [1996] have a finite-sized magnetosphere. When the Alfvèn Mach number is large enough, KH unstable waves are not exponentially decaying surface waves in the magnetosphere but, rather, standing oscillatory waves trapped between the magnetopause and the lower boundary of the magnetosphere. It is natural that the growth rate of such waves depends on the size of the magnetosphere and of its inhomogeneity. In particular, it can be anticipated that the largest growth rate will be for a perturbation with the wavelength of the order of the magnetosphere size, and the numerical results by *Fujita et al.* [1996] support this.

In the case of resonant instability studied in this paper the situation is more complicated than in the case of KH instability. A negative energy wave propagating on the magnetopause is unstable only when the condition of resonance between this wave and local Alfvén oscillations is satisfied at a resonant position in the inhomogeneous region. The growth rate of the negative energy surface wave is proportional to the efficiency of the resonant coupling. The amplitude of the surface wave exponentially decays with the distance from the magnetopause with the characteristic scale k^{-1}. This implies that the amplitudes of short surface waves are exponentially small at the resonant position (which only depends on the phase velocity of the surface wave but not explicitly on its wavelength) and the efficiency of the resonant coupling is very small. As a result, the growth rate of the negative energy surface wave tends to zero when k tends to infinity.

Still, in general, the growth rate takes its maximum value for ka of the order unity (recall that a is the thickness of the inhomogeneous layer). So the main reason why we used the long-wavelength approximation when studying the resonant instability of the magnetopause is mathematical tractability. When ka is of the order unity, only numerical analysis is possible.

However, it turns out that for some unperturbed states, only long negative energy surface waves are unstable, so that the long-wavelength approximation is not so artificial as it seems from first sight. Let us give an example of such an unperturbed state. The resonant position z_A is determined by the condition that at this position the phase velocity of a negative energy surface wave coincides with the local Alfvén speed. The phase velocity of the surface wave depends on ka. We consider the situation where for $ka \to 0$ this phase velocity is close to the minimum value of $v_{A0}(z)$ in the interval $[-a, 0]$, $v_{A0}(0)$. This implies that the resonant condition is satisfied at z_A satisfying $|z_A| \ll a$. When studying dispersion equation (40) using the regular perturbation method, we concentrated on the second term on the right-hand side because it determines the instability increment. However, there is also the first term on the right-hand side. This term provides the dispersion, so that the approximate solution to (40) corresponding to the negative energy wave is

$$\omega = kv_A(M - \beta^{1/2}\xi)\cos\varphi + \kappa ak|k| + i\Im(\omega'), \quad (42)$$

where $\Im(\omega')$ is given by (41). The dispersion coefficient κ can be written in terms of equilibrium quantities; however, here we do not do this. We only note that the important property is that, depending on equilibrium quantities, κ can be either positive or negative. Let us assume that $\kappa < 0$. Then the phase velocity of the negative energy wave decreases when k is increased. The resonance between the surface wave and local Alfvén waves is only possible when this phase phase velocity is in the interval $[v_{A0}(0), v_A]$. Since for $ak \to 0$ the phase velocity is only slighly larger than $v_{A0}(0)$, the resonant condition can be satisfied only when the dispersion correction $\kappa ak|k|$ in (42) is small in comparison with the first term on the right-hand side. Hence the resonant instability takes place only for $ka \ll 1$.

5. It is interesting that the resonant instability has not been found in earlier numerical studies of the magnetopause stability. The reason is that in the majority of numerical works the stability of the equatorial regions of the magnetopause was studied. Consequently, the equilibrium magnetic field is perpendicular to the flow [e.g., *Fujita et al.*, 1996]. Such configurations are KH unstable for any values of the shear velocity, so that there is no KH threshold velocity at all. Since negative energy surface waves can exist only for flow velocities below the KH threshold velocity, there are no negative energy waves and, consequently, no resonant instability in such configurations.

On the other hand, in numerical modeling of the instability of the magnetospheric boundary where the magnetic field and shear velocity were taken to be nonperpendicular, usually only the shear velocity larger than the KH threshold was considered [e.g., *Miura*, 1992]. Hence, once again, the necessary condition for the existence of negative energy surface waves was not satisfied.

7. Conclusions

In this paper we have considered the instability of the magnetopause caused by the resonant interaction of a negative energy surface wave with Alfvén waves localized in the vicinity of the resonant magnetic surface. We have assumed that the magnetosheath plasma is magnetic free and the magnetospheric plasma is cold. In addition, we have made an assumption that the sound speed c_S in the magnetosheath is much smaller than the

Alfvén speed v_A in the magnetosphere and used the ratio c_S/v_A as a small parameter. This assumption is viable, at least for the near magnetotail. We have shown that the threshold for the Kelvin-Helmholtz (KH) instability is close to v_A. Then we considered the surface waves propagating on the magnetopause when the shear velocity is lower than the KH threshold. These waves are the forward wave propagating in the direction of the shear velocity with respect to the magnetosheath plasma and the backward wave propagating in the opposite direction. We have shown that the backward wave is a negative energy wave when its phase velocity is in the direction of the shear velocity with respect to the magnetospheric plasma. This happens when the shear velocity U is larger than the critical velocity U_c, which is of the order c_S. Hence one of the two surface waves is a negative energy wave when $U_c < U < U_{\rm KH}$, $U_c/U_{\rm KH}$ being of the order $c_S/v_A \ll 1$.

When the negative energy surface wave is in resonance with the local Alfvén waves at a resonant surface inside the magnetosphere, it becomes unstable. We study this instability under the assumption that the density grows in the direction toward the magnetopause and, consequently, the Alfvén velocity decreases in a slab with the thickness much smaller than the wavelength. The resonance is only possible when the phase velocity of the negative energy surface wave is larger than the phase velocity of Alfvén waves at the magnetopause. The instability increment is zero when the negative energy surface wave propagates either parallel or perpendicular to the direction of the shear velocity, so that only obliquely propagating surface waves are unstable. The important property of the considered instability is that the amplitudes of Alfvén oscillations in the vicinity of the resonant magnetic surface grow simultaneously with the amplitude of the surface waves.

The manifestation of the resonant instability of the magnetopause can be the correlated growth of the perturbation of the magnetospheric boundary and the Alfvén waves propagating in the magnetotail in the vicinity of the magnetopause and away from Earth.

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