Are two-fluid effects relevant to ULF pulsations?

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Abstract. It has been suggested recently that the traditional single-fluid MHD description of ULF pulsations is wrong and that a two-fluid model (which includes electron inertia) is required [*Bellan*, 1994]. If this claim is correct, it suggests most previous studies are inadequate and of questionable value. We have examined the appropriate equations and find that two-fluid effects are not important for typical ULF pulsations. Specifically, the time taken to develop spatial scales similar to the electron inertia length far exceeds the lifetime of the waves. We argue that the singular normal modes of single-fluid MHD are important properties of the magnetospheric system that should be studied. They may be employed to calculate the evolution of waves and are also a useful mathematical limit of the more realistic dissipative normal modes.

1. Introduction

Single-fluid magnetohydrodynamics (MHD) has been used to describe the behavior of ultralow-frequency (ULF) pulsations for more than four decades [*Dungey*, 1954]. Over the last two decades a variety of detailed analytical and numerical studies have followed the work by *Tamao* [1965], *Southwood* [1974], and *Chen and Hasegawa* [1974]. The single-fluid equations give rise to "resonant" coupling between the fast and Alfvén modes on field lines where the local Alfvén frequency matches the fast frequency. These field lines are sometimes referred to as the Alfvén layer.

Normal mode analysis of the governing ideal single-fluid equations (i.e., solutions proportional to $\exp(-i\omega t)$, where t is time) reveals singular behavior at the resonant field line in the one-dimensional box model [*Southwood*, 1974]. The singular behavior is also found in more general two-dimensional equilibria [*Thompson and Wright*, 1993; *Wright and Thompson*, 1994].

Recently, *Bellan* [1994] has claimed that a two-fluid description of the plasma (i.e., including a finite electron mass) removes the singularity and argues that the waves do not become "resonant" at the Alfvén layer and there is no accumulation of energy there. *Bellan* [1994] asserts that the description of resonant wave coupling based upon single-fluid MHD as employed in laboratory, magnetospheric, and solar plasma physics is erroneous and points out alleged errors in the incompressible and compressible single-fluid MHD, and two-fluid MHD derivations. Bellan's claims have drawn three comments to date which take exception to his assertions [*Goedbloed and Lifschitz*, 1995; *Ruderman et al.*, 1995; *Rauf and Tataronis*, 1995].

The purpose of this report is to examine the relation between single-fluid and two-fluid MHD and to determine the domain of validity of each in relation to typical ULF pulsations. Single-fluid MHD predicts the development with time of ever finer spatial scales. Eventually, such a solution will violate the single-fluid approximations and require a two-fluid treatment. We estimate that typical ULF pulsations on closed field lines ($L<10$) do not live long enough for two-fluid effects to become significant. We conclude that other effects (such as ionospheric dissipation) are more important than electron inertia. The (singular) normal modes of single-fluid MHD remain important properties of a nonuniform medium and are worthy of study and calculation.

2. Models and Approximations

Any model has its domain of validity outside of which it should not be employed without refinement. The most common refinements to ideal, linear, cold, single-fluid MHD are the inclusion of (1) electron inertia (a two-fluid description), (2) kinetic effects, (3) dissipation, (4) plasma pressure, and (5) nonlinear terms. We decide if it is acceptable to ignore these refinements by determining the relative sizes of appropriate terms in the unapproximated equations. If a normal mode corresponded to reality, we would evidently run into problems justifying the linear approximation as the normal mode attains infinite amplitude at the Alfvén layer! Of course, a single normal mode does not correspond to reality [*Ruderman et al.*, 1995] but is a mathematical property of the governing equations.

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Paper number 96JA01947.
0148-0227/96/96JA-01947$09.00

Single-Fluid and Two-Fluid Equations

We shall address the difference between cold, linear, ideal, single-fluid and two-fluid MHD. Let the particles have masses, charges, equilibrium number densities, and fluid velocities of m_s, q_s, n_s, and $\mathbf{u}_s$, respectively ($s = i$ for ions and e for electrons). The macroscopic density ρ, current density $\mathbf{j}$, and center of mass velocity $\mathbf{u}$ are given by

$$\rho = m_i n_i + m_e n_e \quad (1a)$$

$$\mathbf{j} = q_i n_i \mathbf{u}_i + q_e n_e \mathbf{u}_e \quad (1b)$$

$$\rho\mathbf{u} = m_i n_i \mathbf{u}_i + m_e n_e \mathbf{u}_e \quad (1c)$$

Assuming that the ions are singly charged ($q_i = -q_e = q$), the linear center of mass velocity $\mathbf{u}$, current density $\mathbf{j}$, magnetic field $\mathbf{b}$ and electric field $\mathbf{E}$ evolve in the presence of the equilibrium magnetic field $\mathbf{B}$ according to

$$\rho\frac{\partial \mathbf{u}}{\partial t} - \mathbf{j}_\wedge\mathbf{B} = 0 \quad (2a)$$

$$\mathbf{E} + \mathbf{u}_\wedge\mathbf{B} = \frac{m_i m_e}{\rho q^2}\cdot\frac{\partial \mathbf{j}}{\partial t} + \frac{m_i}{\rho q}\left(1 - \frac{m_e}{m_i}\right)\mathbf{j}_\wedge\mathbf{B} \quad (2b)$$

$$\frac{\partial \mathbf{b}}{\partial t} + \nabla_\wedge\mathbf{E} = 0 \quad (2c)$$

$$\nabla_\wedge\mathbf{b} - \mu_0\mathbf{j} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \quad (2d)$$

where c is the speed of light [e.g., *Boyd and Sanderson*, 1969]. The single-fluid, nonrelativistic MHD equations are found by letting $m_e/m_i \to 0$ and neglecting the right-hand sides of (2) compared with the left-hand sides. Thus the relative magnitudes of the right-hand sides give some indication of how important two-fluid effects are.

Bellan [1994] places much emphasis upon the absence of a singularity in the normal modes of the two-fluid equations. For a simple equilibrium containing a uniform magnetic field ($\mathbf{B} = B\hat{\mathbf{z}}$) and with density solely a function of x, we may seek normal modes proportional to $\exp i(k_y y + k_z z - \omega t)$. The governing equations then reduce to ordinary differential equations in x, determining how the modes vary in that direction. Employing the two-fluid equations (2a)-(2d) and eliminating variables in favor of the electric field, we find, after much algebra,

$$ik_y\frac{dE_y}{dx} + ik_z\frac{dE_z}{dx} + \left(k_y^2 + k_z^2 - \frac{\omega^2}{c^2}S\right)E_x + \frac{\omega^2}{c^2}iDE_y = 0 \quad (3a)$$

$$ik_y\frac{dE_x}{dx} - \frac{d^2E_y}{dx^2} - \frac{\omega^2}{c^2}iDE_x + \left(k_z^2 - \frac{\omega^2}{c^2}S\right)E_y - k_y k_z E_z = 0 \quad (3b)$$

$$ik_z\frac{dE_x}{dx} - \frac{d^2E_z}{dx^2} - k_y k_z E_y + \left(k_y^2 - \frac{\omega^2}{c^2}P\right)E_z = 0 \quad (3c)$$

where D, P, and S are functions of x and are defined as

$$D = \omega\cdot\frac{\omega_{ce}\omega_{pe}^2 + \omega_{ci}\omega_{pi}^2}{(\omega^2 - \omega_{ce}^2)(\omega^2 - \omega_{ci}^2)} \quad (4a)$$

$$P = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2} \quad (4b)$$

$$S = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \quad (4c)$$

The plasma and cyclotron frequencies of the species s are defined via

$$\omega_{ps}^2 = \frac{n_s q_s^2}{\epsilon_0 m_s} \quad (5a)$$

$$\omega_{cs} = \frac{q_s B}{m_s} \quad (5b)$$

These equations are identical to the dielectric tensor form given by *Bellan* [1994, equation (35)]. To decide if the equations are singular, we combine the above equations to give a single, fourth-order equation in one variable and see where the coefficient of the highest derivative vanishes. After considerable algebra, (3a)-(3c) may be reduced to the form

$$A_4\frac{d^4E_y}{dx^4} + A_3\frac{d^3E_y}{dx^3} + A_2\frac{d^2E_y}{dx^2} + A_1\frac{dE_y}{dx} + A_0E_y = 0 \quad (6)$$

We have calculated the coefficients in full without omitting any terms, unlike *Bellan* [1994, equations (131) and (132)]. For brevity we give only the explicit form of $A_4(x)$ since we are interested in the existence or otherwise of a singularity in the normal modes.

$$\begin{aligned}A_4(x) = S\Big[&k_y^4\left(P^2 + S^2 - D^2 - 2SP\right) \\ &+ k_y^3\left(PD' - P'D + 3DS' - SD'\right) \\ &+ \frac{\omega^2 k_y^2}{c^2}\left(SD^2 - S^3 - SP^2 + 2PS^2 + PD^2\right) \\ &+ \frac{\omega^2 k_y}{c^2}\left(SDP' - PSD' - 2PDS' - SDS' + S^2D'\right) \\ &+ k_y^2\left(SS'' - 2S'^2 - PS'' + P'S'\right) \\ &+ k_y^2k_z^2(S - P)^2 - \frac{\omega^4}{c^4}PSD^2\Big] \quad (7)\end{aligned}$$

where a prime denotes d/dx.

The coefficent A_4 vanishes when $S(x) = 0$, which corresponds to the upper and lower hybrid resonances [*Stix*, 1992], but it does not vanish where $\omega^2 = k_z^2V_A^2$. (V_A is the Alfvén speed and is equal to $B/\sqrt{\mu_0\rho}$.) Thus *Bellan* [1994] is quite right to claim that the two-fluid normal mode does not contain a singularity at the "Alfvén layer." We have also performed numerical integration of (6) across the Alfvén layer and find regular well-behaved modes in accord with *Bellan* [1994, Figures 2 and 3].

The reader may be curious as to how the singular single-fluid modes arise from (6). We take the limit of low Alfvén speed, $V_A/c \to 0$, and the limit of low frequency, $\omega/\omega_{ci} \to 0$, simultaneously such that the quantity $V_A k/\omega$ remains finite. Finally, we let $m_e \to 0$ so that ω_{pe} becomes infinite and the electron inertia length $\ell_e = c/\omega_{pe}$ becomes vanishingly small. When this limit is taken, the coefficients A_3 and A_4 are smaller than the leading behavior of the other coefficients by a factor V_A^2/c^2. Thus the leading behavior of (6) produces a second order ordinary differential equation (ODE) that has the following coefficients

$$A_4 = A_3 = 0 \quad (8a)$$

$$A_2 = 1 \quad (8b)$$

$$A_1 = \frac{2(\ln V_A)' k_y^2 (\omega/V_A)^2}{(k_z^2 - \omega^2/V_A^2)(k_y^2 + k_z^2 - \omega^2/V_A^2)} \quad (8c)$$

$$A_0 = \frac{\omega^2}{V_A^2} - k_y^2 - k_z^2 \quad (8d)$$

It is easy to confirm that (8a)-(8d) are identical to the familiar single-fluid equation of *Southwood* [1974] and other studies. The normal modes of this equation are well known and contain a singularity at the position where $V_A^2(x) = \omega^2/k_z^2$.

3. Time-Dependent Solutions and Reality

At first sight the vanishingly small scale lengths that exist at the singularity of the single-fluid equations must be smaller than the electron inertia length (since m_e is very small, but not zero) and suggest that the singular modes indicate a violation of the approximations employed in deriving them. As we have already noted the infinite amplitude of the normal mode already indicates that the linear approximation will be violated. These problems are a cause for concern only if you believe that a real system can behave as a solitary normal mode.

A real system is time dependent and never behaves as a single normal mode. Even if the system has been driven for a long time at a single frequency there will still be transients somewhere, although they may be outside the region of interest or of small amplitude. If a real system were described by a single normal mode, we would be forced to conclude that the system had been executing this oscillation for an infinite time, which no real system could be expected to satisfy. Real systems are driven or excited, and this behavior can never be described by a single normal mode. Normal modes do not correspond to reality, but are mathematical functions of the governing equations, and as we shall see, are very useful functions that are well worth studying.

Some confusion may arise from the use of the term "resonance." In a normal mode it is clear what a resonance is. *Rauf and Tataronis* [1995] associate resonant behavior with non-square-integrable normal modes. It is not so clear what the term "resonance" means in a time-dependent system where the meaning of frequency is lost. We would like to offer the following working definition based upon time-dependent behavior and propose two ways of arriving at the result.

Time-Dependent Governing Equations

The most straightforward method is to step back from normal modes and solve the time-dependent equations (2a)-(2d) by numerical or analytical means. Let the system be undriven for $t < 0$ and driven at a single frequency subsequently. (This could be realized by driving a boundary motion at a single frequency or considering a cavity with small k_y, in which case the natural fast modes act as a steady driver.) If the single-fluid equations are employed, we find an accumulation of energy at the Alfvén layer [*Radoski*, 1974; *Allan et al.*, 1986; *Inhester*, 1987; *Lee and Lysak*, 1991; *Wright*, 1992a,b; *Mann et al.*, 1995]. The amplitudes of the fields at the Alfvén layer increase with time, while becoming more localized in the x direction. There is an accumulation of energy at the Alfvén layer; the energy density variation with x has a peak at the Alfvén layer whose amplitude is proportional to t^2 and whose width scales as t^{-1}, giving the energy integrated across the Alfvén layer increasing as t [*Wright*, 1992b]. We shall adopt the increasing amplitude and energy at the Alfvén layer as our working definition of the manifestation of a "resonance" in a time-dependent system.

If the two-fluid equations are solved, similar behavior is observed until times when the solution develops spatial scales that are comparable to the electron inertia length [*Rankin et al.*, 1993; *Wei et al.*, 1994]. At this point, two-fluid effects become important and the first term on the right-hand side of (2b) becomes significant. (The second term on the right-hand side of (2b) is of the order of ω/ω_{ci}, which is independent of the cross-field scale of the wave.) For early times in this solution there is no significant difference between the single-fluid and two-fluid solutions. For large times the two solutions will differ significantly; the single-fluid solution will continue to develop ever finer scales and larger amplitudes at the Alfvén layer, while the two-fluid solution begins to mode convert the Alfvén wave in the "resonant" layer to an inertial electron Alfvén wave which propagates across the background field on the higher Alfvén speed side [*Wei et al.*, 1994; *P. M. Bellan*, Mode conversion into non-MHD waves at the Alfvén layer: The case against the field line resonance concept, submitted to *Journal of Geophysical Research*, 1996]. Thus the two-fluid equations prevent the continual increase in energy at the Alfvén layer at large times. What is important for us is to determine at what point the two-fluid effects become significant for ULF pulsations.

Normal Mode Summation

Although a single normal mode cannot be used to describe the evolution of a real system, a summation or integral of the normal modes can [*Barston*, 1964; *Goedbloed*, 1983; *Cally*, 1991; *Mann et al.*, 1995], and it is for this reason that the modes are so useful. If the single-fluid limit is taken, the normal modes may contain singularities. However, a properly constructed sum of these singular modes yields a solution in (x, t) which is finite and well behaved, with no singularities [see *Barston*, 1964; *Goedbloed*, 1983; *Cally*, 1991; *Mann et al.*, 1995]. Of course, we could work with the two-fluid normal modes and sum these to generate a realistic evolution. The solution resulting from the nonsingular, two-fluid normal modes would be the same as that derived with the single-fluid, singular normal modes until such time that the right-hand side of (2b) becomes significant. At this point the single-fluid approximations start to become invalid and the two solutions will differ. The summed normal mode solutions will give identical results to the integrated causal equations discussed in the previous subsection. Thus accumulation of energy at the Alfvén layer will occur until electron inertia scale lengths are reached, at which point radiation of inertial electron Alfvén waves suppresses further growth. Until then, the development is identical to that of a single-fluid Alfvén "resonance" as we have defined it.

An important point in deciding if a single-fluid description is valid when describing a real system is that the approximations must be checked against the real (or summed normal mode) solution. It should never be applied to a single normal mode. It does not matter if a singular mode has infinite amplitude and vanishing scale length. What matters is if the amplitude and scale lengths of the summed (physically meaningful) solution violate the single-fluid approximations.

4. Spatial Scales

Recently, *Mann et al.* [1995] have shown how time-dependent, ideal, cold plasma solutions develop increasingly fine structure in time. One can think of each field line tending to oscillate with its natural Alfvén frequency, $\omega_A(x)$. As time passes, neighboring field lines drift out of phase with one another and fine spatial scales result. We term the local spatial scale the "phase-mixing length" L_{ph} which is the distance between field lines with the same phase. To an excellent approximation we find

$$L_{ph} = \frac{2\pi}{\omega'_A t} \tag{9}$$

We can invert (9) to define a "phase-mixing time" τ, which is the time it takes for phase mixing to reach some scale L.

$$\tau = \frac{2\pi}{\omega'_A L} \tag{10}$$

Mann et al. [1995] note that these relations can be used to decide when a single-fluid solution will develop scales such as the electron inertia length or ion gyroradius, which would indicate that the solution would cease to be a good approximation to reality. Two-fluid effects begin to become important when spatial scales of the order of the electron inertia length ($\ell_e = c/\omega_{pe}$) are achieved. *Rankin et al.* [1993] and *Wei et al.* [1994] suggest that a scale of 5 or $10\ell_e$ is adequate. We shall follow the suggestion of *Mann et al.* [1995] and estimate the time it will take to develop this scale.

Simulations

To test our predictions we apply our method to the numerical results of *Wei et al.* [1994]. In terms of their time unit T_p, we calculate the phase-mixing time to reach a scale of $10\ell_e$ to be $\tau = 3.4T_p$. *Wei et al.* [1994] ran their simulations twice (once with electron inertia and once without). They then superpose the results and look for differences. *Wei et al.* [1994, Figure 4] display a snapshot at $t = 2T_p$, in which there is no noticeable difference. *Wei et al.* [1994, Figure 5] is a snaphot at $t = 4T_p$, and we would expect some differences to be visible now. Indeed, this is the case. Relatively small but significant differences exist between the two solutions. Further evidence is shown by *Wei et al.* [1994, Figure 7] which displays the two simulation results at times $2T_p$, $4T_p$, $6T_p$, and $8T_p$. Once again, there is no difference at $t = 2T_p$, small but definite differences at $t = 4T_p$, and increasing differences at later times. A crucial requirement for a time-dependent system to exhibit two-fluid effects is the development of fine scales via phase mixing, which is a single-fluid effect!

Magnetosphere

The above analysis shows that our method of deciding when two-fluid effects become significant is reliable. We shall now apply the method to realistic magnetospheric parameters for ULF pulsations that are found on dipole-like field lines. First, we need to choose some representative equilibrium quantities. We take a representative field line to have an invariant latitude of $\Lambda = 67°$ (i.e., $L = 6.5$). The Alfvén frequency and its gradient can be estimated from data. *Poulter et al.* [1984, Figure 2] show that the period at $\Lambda = 67°$ is about 400 s, and the gradient is 100 s per L shell. Thus $\omega_A = 2.5$ mHz and $\omega'_A(\lambda = 0) = 5.7\times10^{-10}$ s^{-1}m^{-1}, where λ is the latitude. The gradient near the ionosphere ($\lambda = \Lambda$) will increase by a factor of 31.5 due to the convergence of field lines. Taking the electron number density at the equator and ionospheric footpoint (at an altitude of 300 km in the F region) to be 10^7 m^{-3} and 10^{11} m^{-3}, respectively, we find the electron inertia lengths are 1.68×10^3 m and 17 m.

Employing (10) we are now able to estimate how many cycles the pulsation must survive before a scale of $10\ell_e$ is reached. We find 2000 cycles are required for the equatorial region and 5 million cycles for the ionospheric end! So the wave must survive for the order of 2000 cycles before we need to concern ourselves with two-fluid effects. ULF data show that pulsations do not live this long but typically survive for 5 to 20 cycles, suggesting that the two-fluid corrections are insignificant.

The situation is complicated somewhat by the variation of plasma density (and thus ℓ_e, which is proportional to $n_e^{-1/2}$) along the field line. Moreover, the separation of L shells decreases with latitude according to the factor

$$s(\lambda) = \frac{\cos^3\lambda}{\sqrt{1 + 3\sin^2\lambda}} \tag{11}$$

The criterion of the phase-mixing length equaling $10\ell_e$ will be satisfied first at a latitude where $\ell_e(\lambda)/s(\lambda)$ is a maximum. Suppose $n_e \propto r^{-q}$ over most of the field line, where the radial distance r may be parameterized in terms of the latitude and equatorial radial distance (r_0) by $r = r_0 \cos^2 \lambda$. For $q > 6$ or less than about 3, two-fluid effects will first become evident at the equatorial or ionospheric sections of the field line, respectively. For intermediate values of q it is easy to show that two-fluid effects initially occur at a latitude λ_e given by

$$\cos^2 \lambda_e = \frac{4(q-3)}{3(q-2)} \qquad (12)$$

For example, if $q = 3.5$ on an $L = 6.5$ field line, two-fluid effects will first manifest themselves at an altitude of 2 R_E. We still need to estimate the number of cycles for phase mixing to reach $10\ell_e$ at this position. Let us choose quantities conducive to the development of two-fluid effects by assuming that there is a strong density depletion at altitude 2 R_E such that $n_e = 5 \times 10^6$ m^{-3}. The wave will now need to phase-mix for 85 cycles, which is still much longer than the lifetime of field line resonances.

5. Discussion and Summary

The fact that standing Alfvén wave pulsations last for 10 or so cycles rather than the infinite lifetime an ideal single-fluid calculation would suggest means that some important ingredient is missing from this model. Of the refinements listed at the beginning of section 2 it is likely that the most significant omissions are dissipation and nonlinear terms. *Newton et al.* [1978] and *Allan and Knox* [1979] show that realistic ionospheric conductivities will give a damping rate γ/ω of order 0.01. Thus the e-folding decay time is about 15 cycles, which is in good agreement with observations. When a realistic value of ionospheric dissipation is included, the smallest scales of the normal modes are restricted [*Wright and Allan*, 1996] and it seems unlikely that the electron inertia length will be reached. *Allan* [1992] and *Tikhonchuk et al.* [1995] have shown that nonlinear effects can also be important and are required to understand the distribution of plasma along field lines.

An ideal, cold, linear description of a plasma neglects many processes. For ULF Alfvén pulsations it seems that the most important omissions are probably dissipative and nonlinear terms. The neglect of two-fluid effects (i.e., electron inertia) is an excellent approximation during the lifetime of typical pulsations found from dawn through noon and dusk. Resonances in the midnight quadrant with extreme plasma depletion in the auroral zone could perhaps develop two-fluid behavior after 85 cycles, which is still several wave lifetimes. It may be possible that some extra ingredient such as nonlinear chaotic phase-mixing could yield two-fluid behavior within the wave's lifetime under extremely favorable conditions, but this has not been demonstrated. While two-fluid effects are not important for the type of ULF pulsations described above, we note that *Streltsov and Lotko* [1995] suggest electron inertial and kinetic effects may become significant in thin layers around isolated density enhancements where the gradient of the Alfvén frequency may be much larger than the value adopted in our equilibrium magnetosphere. Indeed, we do not claim that ULF pulsations will never have scales small enough that two-fluid effects are important. We are merely pointing out that these scales will not be achieved through "resonant" linear MHD wave coupling for realistic timescales. Thus some other effect must be responsible for producing small scales when (or if) they exist.

We conclude that two-fluid corrections to the traditional single-fluid analysis are not of importance in the ULF resonant coupling model proposed by *Southwood* [1974] and *Chen and Hasegawa* [1974] for typical magnetospheric conditions. The singular normal modes of these studies are certainly worthy of calculation and indicate where resonances will grow in real time-dependent systems. When dissipative effects are introduced to a single-fluid analysis the arguments of the ideal, singular functions acquire a small imaginary component which removes the singular behavior. Considerable insight and understanding of resonant wave coupling have been achieved through studying the singular normal modes and their relation to regular dissipative normal modes within the single-fluid MHD limit.

Acknowledgments. A.N.W. is supported through a U.K. PPARC Advanced Fellowship and is grateful to PPARC and the New Zealand Foundation for Research, Science and Technology (FRST) for funding his visit to NIWA, New Zealand. W.A. received support from FRST under contract CO1309.

The Editor thanks John S. Samson, Marcel Goossens, and Michael Ruderman for their assistance in evaluating this paper.

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(Received February 7, 1996; revised June 11, 1996; accepted June 17, 1996.)