THE EFFECT OF RECONNECTION UPON THE LINKAGE AND INTERIOR STRUCTURE OF MAGNETIC FLUX TUBES

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Abstract. The topological properties of a magnetic field are interpreted in terms of magnetic helicity. The total helicity of a collection of flux tubes arises from the linking of flux tubes with one another (mutual helicity) and the internal magnetic structure of each flux tube (self-helicity). Reconnection changes the topology and magnetic connectivity of flux tubes. This can also be viewed as a redistribution of self- and mutual helicities, If total magnetic helicity is approximately conserved, it is possible to put quantitative limits upon the changes in self- and mutual helicities. This can be interpreted as the change in magnetic flux tube linkage (due to reconnection) and amount of twist present in the reconnected flux tubes. The implications for reconnection in the terrestrial magnetosphere are also discussed.

1. Introduction

There are many systems in nature where topological properties are important. For example, divergence-free vector fields (vortex lines, incompressible flow lines, magnetic field lines) have no end points and often form closed loops. The topology of curves has also been applied to polymer chains and the structure of DNA [see Berger and Field, 1984, and references therein]. In a recent study [Berger and Field, 1984] the properties of a magnetic field have been investigated in terms of magnetic helicity. By dividing space into magnetic flux tubes they presented a formalism with which to calculate the helicity of a collection of flux tubes (see Berger [1988] also). The total helicity of a system can be thought of in terms of two contributions: There is "mutual helicity" due to linking or knotting of different flux tubes with one another, and there is "self-helicity" which arises from the internal structure of each flux tube (i.e., twisting and kinking). It is interesting to note the related concepts used in mathematical biology of "writhing" and "twist" numbers.

The magnetic helicity is dissipated in a resistive plasma. However, Taylor [1974] has suggested that the net dissipation will be small on reconnection time scales, in high magnetic Reynolds number plasmas. This suggestion has been proven analytically by Berger [1984] (see Appendix A). Reconnection allows the magnetic field to diffuse and thus change its topology and linkage. If helicity is approximately conserved, reconnection would redistribute the total helicity between self- and mutual helicities. For example, if reconnection were to unlink two flux tubes (i.e., decrease the amount of mutual helicity), the new magnetic flux tubes may be twisted (i.e., increased self-helicity). This concept forms the

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Paper number 88JA03839. 0148-0227/89/88JA-03839\$05.00 foundation of this paper. Two sections of this paper present worked calculations of the effect of reconnection upon the topology and distribution of self- and mutual helicities.

The best studied examples of reconnection in nature occur in the terrestrial magnetosphere. Dungey [1961] suggested that on the Earth's dayside, solar wind and geomagnetic fields would reconnect and produce open magnetic field lines. These open field lines move to the Earth's nightside where they form a tail and can reconnect with each other. There has been considerable detail added to the open magnetosphere model in recent years. In particular, the reconnection process on both the dayside and nightside is thought to have a sporadic and transient nature. On the dayside, isolated open flux tubes are observed frequently [Russell and Elphic, 1978]. These open flux tubes have a twisted interior [Cowley, 1982; Paschmann et al., 1982; Saunders et al., 1984], and there has been considerable effort over the last few years to explain the origin of this twist [Lee and Fu, 1985; Sato et al., 1986; Sonnerup, 1987; Wright, 1987].

Reconnection on the nightside is thought to occur in a quasi-steady fashion about 60 Earth radii away from Earth. There is also some sporadic reconnection much nearer to the Earth (15 R_E). The transient reconnection is associated with substorms and produces plasmoids (closed magnetic field loops) between the two reconnection sites [Hones, 1984]. Twisted magnetic field structures have been observed on the nightside [Sibeck et al., 1984; Elphic et al., 1986]. In the discussion we compare our topological notions with observations and models of dayside and nightside reconnection.

The paper is structured in the following fashion: section 2 presents useful results for calculating the magnetic helicity of flux tubes; section 3 examines how reconnection affects the distribution of mutual and self-helicities in a simple system of flux tubes; section 4 considers more realistic reconnection systems that are governed by prescribed reconnection lines; section 5 discusses the results of the preceding sections and the implications for reconnection 6 summarizes the main points of our work and concludes the paper. Appendix A gives the criteria for small helicity dissipation, and Appendix B discusses the partition of self-helicity between two reconnected fluxes.

2. Relation of Magnetic Helicity to Field Topology

Helicity integrals provide a natural measure of topological structure in a vector field [Moffatt, 1969; Berger and Field, 1984]. We wish to discuss the relation of helicity to topology for fields undergoing reconnection. In order to concentrate on structure near the reconnection site, we restrict the helicity integral to a subvolume of space v containing the site. For simplicity this subvolume will be the space between two parallel planes.

These planes have no physical significance; plasma and fields can pass through them freely. The planes are chosen to be perpendicular to the mean magnetic field.

Let B be the magnetic field, A its vector potential, and B_0 the vacuum field $(\nabla_A B_0=0)$ sharing the same normal components at the boundary planes $(B.\hat{n}=B_0.\hat{n})$. Choose a vector potential A_0 for the vacuum field whose transverse components at the boundaries coincide with A $(A_A \hat{n}=A_{0A} \hat{n})$. Then the gauge invariant form of magnetic helicity (the "relative helicity") can be written [Berger and Field, 1984; Jensen and Chu, 1984]

$$H=\int_{U}(\mathbf{A}.\mathbf{B}-\mathbf{A}_{n}.\mathbf{B}_{n})d^{3}r$$
 (1)

First consider a single flux tube extending between the two planes. If this tube were axisymmetric and uniformly twisted, then $H=T\Phi^2$, where Φ is the flux within the tube and T is the net angle (divided by 2π) that a field line twists about the central axis while traveling between the two planes. For a nonuniformly twisted tube, let $\Phi(r)$ be the flux within radius r, T(r) the twist of field lines at r, and R the maximum radius. Then [Kruskal and Kulsrud, 1958; Berger and Field, 1984]

$$\int_{0}^{R} T(r)\Phi(r)(d\Phi/dr)dr$$
 (2)

The helicity of more general fields can be expressed as a sum of "winding numbers" or "linking numbers" θ_{12} between individual lines [Berger, 1988]. This will prove useful in our calculations. We suppose all field lines extend between both planes (say z=0 and z=L). Consider two field lines described by the curves $(x_1(z),y_1(z))$, $(x_2(z),y_2(z))$. (See Figure 1.) At z=0, these curves have endpoints $\mathbf{r}_1 = (x_1(0),y_1(0))$, $\mathbf{r}_2 = (x_2(0),y_2(0))$. At each z, consider the line segment between the two points $(x_1(z),y_1(z))$, $(x_2(z),y_2(z))$. This line segment rotates by



Fig. 1. (a) The field lines with endpoints $r_1(0)$ and $r_2(0)$ at z=0 rotate about each other by a net angle $\theta_{12}=\varphi(L)-\varphi(0)$. (b) After reconnection the angle between the endpoints at z=L becomes $\varphi'(L)=\varphi(L)-\pi$, and so $\theta_{12}'=\theta_{12}-\pi$.

some net amount θ_{12} in going from z=0 to z=L (in other words, the field lines twine about each other by a net angle θ_{12}). Suppose we replace the field lines in Figure 1a by two tubes, each of flux Φ . If these are thin tubes (compared to the distance $|\mathbf{r}_1 - \mathbf{r}_2|$), θ_{12} is roughly constant between fieldlines inside the two tubes.

$$H_{12} = \iint_{Z=0}^{(\theta_{12}/2\pi)} B_{Z}(r_{1}) B_{Z}(r_{2}) d^{2}r_{1} d^{2}r_{2} \qquad (3)$$

Here \mathbf{r}_1 ranges over the endpoints of field lines in tube 1, and similarly for \mathbf{r}_2 and tube 2. If the two tubes are internally untwisted, the total helicity $H=2H_{12}=2(\theta_{12}/2\pi)\Phi^2$. Often one may wish to subdivide the field into a collection of flux tubes. In this case the helicity may be expressed as a sum of self-helicities H_i arising from internal structure or twist within tube *i*, plus a sum of mutual helicities H_{ij} arising from the braiding of tubes about each other. We can write this for N tubes:

$$H = \Sigma H_i + \sum_{i \neq j} H_{ij}$$
(4)

As an example, a bundle of flux whose total helicity is not large may yet contain filaments which are highly twisted. Consider N parallel, cylindrical flux tubes each with twist T and flux Φ/N . The total helicity is H = $NT(\Phi/N)^2 = (T/N)\Phi^2$. Thus this configuration has the same helicity as a single flux tube of flux Φ , but with a twist of only T/N. (One might note that the latter configuration has considerably less magnetic energy.)

What happens when we reconnect field lines? Again, regard Figure 1a as showing two flux tubes crossing each other. The total helicity is

$$H = H_1 + H_2 + 2H_{12}$$
(5)

and $H_{1,2} = (\theta_{1,2}/2\pi)\Phi^2$. If the tubes reconnect, as in Figure 1b, the mutual helicity is reduced to

$$H'_{12} = \left[\theta_{12}/(2\pi) - \frac{1}{2}\right]\Phi^2 \tag{6}$$

As discussed in Appendix A, magnetic helicity conservation is an excellent approximation during rapid reconnection. Thus for the reconnection in Figure 1, the self-helicities must increase by one unit:

$$H'_1 + H'_2 = H_1 + H_2 + \Phi^2$$
 (7)

Appendix B shows that under fairly general conditions (no creation of crossover/anticrossover pairs; these terms will be defined below) the extra self-helicity produced will be partitioned between the two flux tubes equally. Thus $H_1^{+}=H_1+\frac{1}{2}\Phi^2$ and $H_2^{+}=H_2+\frac{1}{2}\Phi^2$. Define the mean twist of flux tube 1 by $\tau_1=H_1/\Phi^2$, and similarly for tube 2. Then both τ_1 and τ_2 increase by $\frac{1}{2}$ (half a complete uniform twist) during reconnection.

Figure 1 has been drawn so that $\theta_{12} \approx \pi$. As seen in projection, the two lines cross over each other. Of course, if $\theta_{12} < \pi$, then there would be projection angles where no crossovers exist. However, for our purposes we will always draw our lines so that $\theta_{12}=\pm\pi$. This simplifies the interpretation of the diagrams without creating any essential difficulties. We are interested in the transfer of mutual helicity to self-helicity; this transfer depends upon the change in θ_{12} due to reconnection. As shown in Figure 1, θ_{12} changes by exactly π (or $-\pi$). Crossovers can be either positive or negative; but if tube 1 went behind tube 2 at the crossover rather than in front, then

the cross over would be negative (and $\theta_{12} \approx -\pi$). Removal of an anticrossover adds a negative (left-handed) half twist to each flux tube involved.

For the remainder of this paper we shall only use the following two results; the mutual helicity due to a crossover $(\theta_{12} \approx \pm \pi)$ of fluxes Φ_1 and Φ_2 is $\pm \Phi_1 \Phi_2$; a tube of flux Φ that has a uniform twist T has a self-helicity of $T\Phi^2$.

3. Simple Flux Tube Systems

In this section we shall discuss some possible reconnection topologies that can be achieved with four flux tubes, each of flux Φ_1 . Throughout this discussion we shall assume that the flux tubes are initially long and straight and cross one another in such a fashion that $\theta_{ij} = +\pi$ (positive crossover) or $\theta_{ij} = 0$ (coplanar, perhaps parallel, flux tubes). The initial state is shown in Figure 2a. The arrows denote the direction of the magnetic field. In order to discuss the field geometry we shall introduce the following nomenclature: "Upper" and "lower" refer to the top and bottom half of the figure, respectively. Flux entering or exiting the central region of the figure will be termed "incoming" or "outgoing," depending upon the direction of the magnetic field. Finally, we shall refer to a collection of flux tubes as a "flux bundle." Thus Figure 2a consists of four flux tubes of flux Φ_1 which can be viewed as two flux bundles of flux $\Phi_0 = 2\dot{\Phi}_1$. The upper incoming flux bundle (top left) maps entirely to the lower outgoing flux bundle (bottom right), and the lower incoming flux bundle (bottom left) maps entirely to the upper outgoing (top right) one. All of the flux tubes (and flux bundles) are initially untwisted, and there is no self-helicity. The total helicity of the system is entirely mutual helicity, due to the crossover of fluxes. The total helicity can be calculated either as a single crossover of a flux bundle (Φ_0) with a similar flux bundle, or equivalently as four crossovers of flux $\Phi_1 = \frac{1}{2}\Phi_0$ with each other. This yields a total helicity of

$$\mathbf{H} = 4\Phi_1^2 \tag{8}$$

In Figure 2b each flux tube has been reconnected once at the reconnection line indicated in bold. Reconnection has destroyed two crossovers (each of Φ_1 with Φ_1) which produced a mutual helicity of $2\Phi_1^2$. The total helicity (8) is now composed of a mutual helicity contribution $(2\Phi_1^2)$ and also some self-helicity $(2\Phi_1^2)$. Appendix B shows how the new self-helicity is shared equally between reconnected fluxes, and so each tube Φ_1 has a self-helicity of $\frac{1}{2}\Phi_1^2$. This could be due to a positive half twist in each tube. (The detailed structure of the interior of each flux tube will depend upon whether there is additional reconnection between field lines in the same flux tube; see below.)

The two remaining crossovers in Figure 2b have been removed by reconnection at two new reconnection lines in Figure 2c. Now there are no crossovers at all, and the total helicity (8) is the sum of self-helicities. If we interpret self-helicity as being due to a uniform twist, the following description is evident: The uppermost and lowest flux tubes will both have a half twist each, and the two central flux tubes both have one and a half twists.

Figure 2 can also be viewed as two bundles of flux (Φ_0) which have a positive crossover in Figure 2a, and no crossovers in Figures 2b and 2c. Therefore the self-helicities of both the flux bundles in Figures 2b and 2c are $\frac{1}{2}\Phi_0^2$. Consider only the upper flux bundle: In Figure 2b it is composed of two flux tubes (each of flux $\Phi_1 = \frac{1}{2}\Phi_0$) that have a half twist and wrap around each other by π (a positive crossover). In Figure 2c the upper



Fig. 2. (a) Four untwisted flux tubes. These have undergone (b) single neutral line reconnection and (c) triple neutral line reconnection. With multiple neutral lines it is possible to produce flux tubes with more than a half twist. (The arrows denote the direction of the magnetic field.)

flux bundle has the same self-helicity as that in Figure 2b, but it is distributed among its constituent flux tubes in a different manner. Additional reconnection has removed the positive crossover of the flux tubes that comprise the flux bundle, so that they do not twine about one another. Instead they have an increased twist. This is another example of how the self-helicity from a

given flux can be composed of detailed structure within flux elements, rather than a uniform twist. In particular, a flux that has a self-helicity equivalent to a uniform half twist may have some elements of flux that have significantly more than a half twist (cf. Figure 2c).

4. Multiple Magnetic Reconnection Lines

In the previous section we discussed the reconnection of discrete flux tubes to one another. This approach will not be continued here, but we shall generalize the description to continuous flux. Throughout this section we shall assume that reconnection occurs along prescribed reconnection lines in a uniform fashion, and for a specific duration. Figure 3a shows the situation that will be produced given two neutral lines, both of length D. (We have only shown the fluxes that have undergone reconnection, since the contribution to total helicity from any other flux is the same before and after reconnection.) Before reconnection, the system consisted of a flux Φ_0 ($\Phi_0=\Phi_1+\Phi_2$) crossing over another flux Φ_0 . After reconnection the system is somewhat more complicated. The fluxes can be divided using separatrices to illustrate the topology more clearly. The upper incoming flux bundle (Φ_0) is composed of a flux tube Φ_1 , which is only reconnected once, and a flux tube Φ_2 , which may undergo reconnection more than once. If the angle of inclination of the magnetic field to the neutral lines is β and these lines are a distance d apart, then



Fig. 3. Reconnection of continuous flux along prescribed neutral lines: The bold horizontal lines represent two reconnection lines. The flux that is reconnected by these lines is bounded by the solid and dashed separatrices. (The solid ones lie above the dashed ones.) The arrows denote the direction of the magnetic field, which forms an angle β to the reconnection lines. (a) The two neutral lines both have a length D and separation d. Thus the flux entering and leaving the shaded region is divided equally between the upper and lower magnetic fields. (b) The lower neutral line has a length D-dcot β . This produces an asymmetry in the way flux leaves the region between the neutral lines.

$$\Phi_1 = \Phi_2(D/d) \tan\beta \tag{9}$$

The length along the neutral lines for a half twist of the magnetic field lines in the shaded region of Figure 3a is $1=\operatorname{dcot}\beta$. If D=nl (n integer), then the incoming flux Φ_2 from the upper bundle maps to the outgoing flux Φ_2 in the upper/lower bundle for n odd/even (similarly for the incoming flux Φ_2 in the lower flux bundle). If d≠nl, then each outgoing flux Φ_2 is composed partially of the upper incoming flux Φ_2 , and the lower incoming flux Φ_2 .

The total helicity before reconnection is due to a crossover of $\Phi_0=\Phi_1+\Phi_2$ with Φ_0 ,

$$\mathbf{H} = \Phi_0^2 \tag{10}$$

After reconnection, the two fluxes Φ_1 each have a half twist, and no crossovers. This contributes $2.(\frac{1}{2}\Phi_1^2)$ to the total helicity. There are two obvious methods to calculate the helicity due to the Φ_2 fluxes. One possibility would be to divide the Φ_2 fluxes into four elements. For example, the upper incoming flux Φ_2 would be divided into flux that emerged in the upper outgoing flux Φ_2 and flux that emerged in the lower outgoing flux Φ_{2} , etc. The position of these separatrices can be determined by mapping the endpoints of the reconnection lines along the magnetic field lines through the shaded area in Figure 3a. This would decompose the fluxes Φ_2 into four flux elements, from which it would be easy to calculate the self- and mutual helicities. The problem with this approach is that the separatrices are dependent upon the position of the ends of the neutral lines and their separation; this necessitates some tedious algebra.

A simpler description of the fluxes Φ_2 is achieved by considering the sections of flux outside the neutral lines separately from those inside the neutral lines (i.e., in the shaded region). The combined exterior sections constitute a crossover of flux Φ_2 with Φ_2 . This gives rise to a helicity of Φ_2^2 . The only fluxes whose helicity we have not calculated are those in the shaded region, between the neutral lines. From conservation of total helicity, we can anticipate that this volume contributes $2\Phi_2^2(D/d)\tan\beta$. This is exactly what we would expect for a tube of flux $2\Phi_2$ that has a uniform twist $T=\frac{1}{2}(D/d)\tan\beta$ (given the pitch length above). This is probably the most straightforward way to view the flux between the neutral lines.

If the ends of both reconnection lines are level, then the flux entering or exiting from that end will be distributed equally between the upper and lower flux bundles. This is not true if the neutral lines are not level. Figure 3b shows an extreme case where all of the outgoing flux from between the neutral lines lies in the lower outgoing flux bundle. (This requires that the length of the lower reconnection line is $D-dcot\beta$.) It is also possible to generate configurations somewhere between the two examples in Figures 3a and 3b by choosing a suitable overlap, as we shall see below.

The total helicity of the system shown in Figure 3b before reconnection is simply due to the crossover of Φ_1 with $\Phi_1 + \Phi_2$; H= $\Phi_1(\Phi_1 + \Phi_2)$. After reconnection, the upper tube of flux Φ , has a half twist and contributes $\frac{1}{2}\Phi_1^2$ to the total helicity. Similarly the lower tube of flux $(\Phi_1 - \Phi_2)$ contributes a self-helicity of $\frac{1}{2}(\Phi_1 - \Phi_2)^2$. The remaining fluxes are best considered in terms of the sections inside and outside the shaded region shown in Figure 3b. The sections outside this region constitute a flux of Φ_2 crossing another flux of Φ_2 that has a half twist (this produces a helicity of $\Phi_2^2 + \frac{1}{2}\Phi_2^2$). Conservation of total helicity requires that the helicity of the shaded region be $2\Phi_{2}^{2}[(D/d)\tan\beta - 1]$. Again, this is consistent with what is expected for a tube of flux $2\Phi_2$ that has a pitch 2dcotβ length and length $(D-dcot\beta),$ i.e.. $T=\frac{1}{2}(D/d)\tan\beta-\frac{1}{2}$.

The properties of flux that is reconnected (at a uniform rate) by two parallel reconnection lines can be characterized by the overshoot at the ends and the overlap in the middle. For example, the overlap (ΔD_1) is zero for both left and right ends in Figure 3a. In Figure 3b the left-hand overlap is again zero, but the right-hand one is $\Delta D_1 = -d \cot \beta$. (We shall use the convention that ΔD_1 is positive if the lower neutral line extends beyond the upper one, and ΔD_1 is negative if the upper neutral line extends beyond the lower one.) It is convenient to use the angle δ to classify the way that flux enters or exits a given end. The parameter δ is defined by $\tan \delta = \Delta D_1/d$. If $|\delta| > \pi/2 - \beta$, the flux from that end maps entirely to the upper/lower flux bundle for δ positive/negative. If $|\delta| < \pi/2 - \beta$, the flux at that end maps to both the upper and lower flux bundles. The fraction of flux mapping to the upper bundle is $\frac{1}{2}(1+\tan\beta\tan\delta)$, and the fraction mapping to the lower bundle is $\frac{1}{2}(1-\tan\beta\tan\delta)$. The overlap of the reconnection lines (ΔD_2) is simply the length of the shaded regions in Figures 3a and 3b. The flux in this region will have a twist of $T = \frac{1}{2} \tan \beta \Delta D_2/d$.

The extension of this type of analysis to several neutral lines is obvious. However, maintaining a general description becomes rather cumbersome for even three neutral lines as the position of each neutral line end relative to the others can change the field configuration dramatically. Further generalization, by allowing nonuniform reconnection rates, produces an even wider variety of magnetic field topologies that can be described with the techniques presented here. We shall only consider a particularly simple limit of these generalizations: Suppose that reconnection occurs over a finite area via a fine tearing mode whose island spacing is much less than the width of the incoming flux bundle. Furthermore, we shall assume that the reconnection rate varies across this area such that it is greatest in the center, and decreases toward the edges. The upper incoming flux bundle would map, almost entirely, to the upper outgoing flux bundle (similarly for the lower flux bundles). Although each reconnected flux bundle would have a self-helicity coresponding to a uniform half twist, it would be composed of filaments that could have many more times this twist (cf. section 2). A similar configuration could be obtained by the magnetic field percolation process described by Galeev et al. [1986]. (This mechanism allows field lines to wander stochastically rather than be confined to a magnetic island.) It is rather difficult to imagine the topology of these field lines. A crude picture can be obtained by allowing, say, one half of the self-helicity of the reconnected tube to be due to a uniform one-quarter twist. If the tube is then considered in terms of N flux elements (cf. section 2), each of these elements could have a twist T=N/4. In reality the helicity would probably arise from magnetic fields over a continuous spectrum of spatial scales.

5. Discussion

In the preceding sections we have developed a formalism that is able to put quantitative limits upon the amount of twist produced in magnetic flux tubes by a change in magnetic flux tube linkage. This description relies upon the approximate conservation of magnetic helicity, as discussed in Appendix A. We shall now apply the results of the preceding sections to magnetospheric reconnection.

Dayside Reconnection

Reconnection between the terrestrial and interplanetary magnetic fields is thought to occur on the Earth's dayside [Dungey, 1961]. This process can take place in a

quasi-steady or sporadic fashion. We shall only discuss the latter here. Spacecraft observations suggest that isolated reconnected flux tubes are formed during transient reconnection [Russell and Elphic, 1978]. These reconnected flux tubes move poleward [Rijnbeek et al., 1984] and appear to have a twisted interior [Cowley, 1982; Paschmann et al., 1982]. The flux transfer events (FTEs) studied by Saunders et al. [1984] had a twisted interior that appeared to be a propagating torsional Alfvén wave. Recently Rijnbeek et al. [1987] have reported some small-amplitude field fluctuations about the ambient interior field twist. They also identified a transition region between the reconnected and background fields.

The original model of Russell and Elphic envisaged reconnection producing a transition from a configuration like that in Figure 1a to one like that in Figure 1b. However, it was not realized until recently that these reconnected flux tubes would indeed be twisted [Wright, 1987]. Wright pointed out (using a qualitative description of the reconnection site) that these flux tubes would both have a half twist. The present paper gives this statement a firm quantitative footing. Wright [1987] also showed how the sections of tube leaving the reconnection region could have the twist distributed along them via torsional Alfvén waves, in agreement with the observations of Saunders et al. [1984]. If twist is produced in this fashion during FTEs, then there can be no more than a half twist in each reconnected flux tube. The Russell and Elphic scenario requires pairwise formation of northern and southern reconnected flux tubes. These tubes may move poleward due to field line tension.

Most other FTE models have invoked multiple reconnection lines. This has the advantage that some elements of reconnected flux can have much more than a half twist (see Figure 2c). Lee and Fu [1985] have suggested that reconnection is likely to occur via the tearing mode. In their view, the magnetic islands between the neutral lines are the twisted flux tubes observed by spacecraft. This means that the number of FTEs produced is equal to the number of magnetic islands. The tearing mode will not necessarily produce FTEs in a pairwise fashion (cf. Figure 3). The twisted flux tube shown in Figure 3a does not experience any net magnetic force up or down. The motion of this tube will probably be determined by the background plasma flow. Lee and Fu [1985] assumed that reconnection would occur via three neutral lines. This yields pairwise FTEs. More recent simulations [Fu and Lee, 1986] have also demonstrated this type of evolution. However, these simulations imposed symmetry requirements that encourage the formation of two magnetic islands [Fu and Lee, 1986; Figures 2 and 8]. When this symmetry is not imposed, the system does not evolve into two islands [Fu and Lee, 1986; Figure 12]. These simulations are two dimensional, and so cannot address the question of magnetic connectivity at the ends of the neutral lines (i.e., where does the incoming flux originate, and the outgoing flux terminate?). The situation described qualitatively by Lee and Fu [1985] consisted of a separate northern and southern flux tube with a connectivity reminiscent of that described by Russell and Elphic [1978]. This would require an overlap of two adjacent neutral lines like that shown on the right-hand side of Figure 3b. In practice it would be sufficient that the majority of the flux comprising the twisted flux tube entered/exited the reconnection region in one bundle. However, it seems unlikely that even this criterion would be met with any consistency in nature. If a given FTE did satisfy this condition, then the twist residing between the neutral lines could redistribute itself along the incoming/outgoing flux tube via torsional Alfvén waves in the fashion described by Wright [1987].

It has been suggested by Galeev et al. [1986] that reconnection may not occur via single or multiple neutral lines. Instead, magnetic field lines wander stochastically across the current sheet. This percolation process could produce a great deal of structure inside the reconnected flux, as could a fine tearing mode. Rijnbeek et al. [1987] have reported small-scale magnetic fluctuations on reconnected field lines. It is not clear whether the field oscillation has a spatial or temporal origin. The reconnection process described by Galeev et al. [1986], or a fine tearing mode, could provide a spatial variation that would produce field fluctuations like those reported by Rijnbeek et al. [1987].

Nightside Reconnection

The open flux tubes formed by dayside reconnection form an elongated tail on the nightside. There is some quasi-steady reconnection at a distant neutral line (about 60 R_E from Earth), and transient reconnection much closer to Earth (about 15 R_E). If the solar wind magnetic field has no dawn-dusk component (IMF $B_v=0$), the tail is symmetric and the northern and southern flux tubes are antiparallel to one another. (See Figures 3a and 3b when $\beta \rightarrow \pi/2$.) Before any flux is reconnected, the magnetic helicity of the tail is zero. Hones [1984] has described the geometry of the magnetic field after reconnection; closed field lines are produced earthward of the near-Earth neutral line; closed magnetic loops are formed between the two reconnection sites; field lines with both ends in the solar wind lie beyond the distant neutral line. The region occupied by closed magnetic loops is called a plasmoid. It is eventually expelled down the tail due to increased reconnection at the near-Earth neutral line that envelops the plasmoid in field lines connected to the solar wind. (See Hones [1984] and references therein.)

There have also been observations of flux ropes (twisted flux tubes) in the geomagnetic tail. Elphic et al. [1986] reported flux ropes in the near-Earth tail (20 R_E) that had a dawn-dusk alignment. Sibeck et al. [1984] observed flux ropes much deeper in the tail (100 R_E) that were oriented antisunward. To produce such magnetic structures it is likely that IMF $B_{\psi}\neq0$, so that the fields in the tail are not antiparallel ($\beta\neq\pi/2$ in Figures 3a and 3b). The near-Earth flux ropes could be produced by the formation of two or more near-Earth neutral lines, or a "nightside FTE," as suggested by Elphic et al. [1986].

Recently Hughes and Sibeck [1987] have considered the three-dimensional structure of plasmoids. They presented data that show that the dawn-dusk magnetic field in the tail does indeed correlate with the same component in the solar wind. The magnetic connectivity of plasmoids was also discussed by Hughes and Sibeck [1987]. They suggest that the rope (whose axis was oriented antisunward) would evolve naturally from a flux rope that has field entering one end from solely the solar wind and exiting the other end exclusively to Earth. The flux rope observed by Sibeck et al. [1984] could also be produced by a configuration like that shown in Figures 3a and 3b if there were a strong enhancement of reconnection at the near-Earth neutral line. This would cause field lines (with both ends in the solar wind) to drape over the flux rope and exert a magnetic force antisunward. If this force is distributed unevenly along the length of the flux rope, it will encourage the rope to bend and distort (especially if it is connected at either end to the Earth) (see Figure 2 of Hughes and Sibeck [1987]). The stretched and distorted tube will naturally evolve sections where the tube axis points antisunward. Alternatively, the flux ropes reported by Sibeck et al. [1984] could be produced by an uneven overlap of near and distant neutral lines (Figure 3b). The twisted section (that lies across the current sheet) can unravel itself along the flux tube that lies approximately parallel to the tail [cf. Wright, 1987].

6. Conclusions

We have presented a description of the topology, or magnetic linkage, of a system of flux tubes in terms of magnetic helicity. We have used results from Berger and Field [1984] and Berger [1988] that show how helicity arises from knotting and linking of individual flux tubes with one another (mutual helicity), and also from the twisting and kinking of each flux tube (self-helicity). Reconnection changes the topology and internal structure of flux tubes, and we have been able to put quantitative limits upon the redistribution of self- and mutual helicities during reconnection. Our formalism has been used to analyze the current models that describe reconnection in the Earth's magnetosphere. It is not obvious that any particular model is right or wrong. Indeed, it may be the case that they are all important, under the appropriate conditions. The main properties of the models are as follows. Single neutral line reconnection produces reconnected flux tubes in a pairwise fashion. Each tube has a half twist and a simple magnetic connectivity. Multiple neutral line models may produce elements of magnetic flux with significantly more than a half twist. The number of flux ropes formed is dependent upon the number of neutral lines and their positions; there need not be pairwise production. The magnetic connectivity of each flux rope is complex. In general both incoming and outgoing flux (for each flux rope) will be composed of solar and terrestrial magnetic fields.

The general formalism that we have presented may be useful in modeling many other situations where reconnection is important, e.g., solar corona, pulsar magnetospheres, and plasma fusion.

Appendix A

The arguments in this paper assume that magnetic helicity dissipation during reconnection is negligible. This assumption can be proven rigorously in the limit of high magnetic Reynolds number [Berger, 1984; Boozer, 1986]. Here we briefly discuss the situation of magnetospheric reconnection.

For simplicity, assume a uniform (or mean) resistivity η . The magnetic energy dissipation rate (dW/dt) and helicity dissipation rate (dH/dt) go as

$$dW/dt \approx - [\eta_j^2 dv \quad dH/dt \approx -2[\eta_j] \cdot Bdv \quad (A1)$$

A Schwarz inequality relates dH/dt to dW/dt, and $\int \eta B^2 dv = 2\mu_0 \eta' W$. (W is the magnetic energy, and η' is the averaged resistivity: $\eta' = \int \eta B^2 dv / \int B^2 dv$.) One finds

$$(dH/dt)^2 \leq -8\mu_0\eta' W(dW/dt)$$
(A2)

Suppose a reconnection event takes place over a time Δt . Then the maximum helicity dissipation is

$$(\Delta H)^{2} \leq 4\mu_{0}\eta'\Delta t (W_{i}^{2}-W_{f}^{2})$$
(A3)

where W_i is the energy before reconnection and W_f the final energy.

Let us look at the reconnection of two bundles of flux as in Figure 1. The regions where the tubes cross the planes have an area A and normal field strength B_z . Before reconnection the field lines have an angle γ with respect to the 2 direction. The initial magnetic energy of both tubes is $W_i = LAsec(\gamma)B_Z^2/\mu_0$. After reconnection the tubes are parallel to 2, with possibly some internal twist. The final energy is greater than that of two untwisted flux tubes, $W_f > LAcos(\gamma)B_Z^2/\mu_0$. The initial helicity is $H \approx \Phi^2$. Thus equation (A3) gives, after some algebra,

$$\Delta H/H < 2.(\eta' \Delta t/\mu_{\eta})^{\frac{1}{2}}.(L/A).(\sec^{2}\gamma - \cos^{2}\gamma)^{\frac{1}{2}}$$
(A4)

The classical resistivity is given by $\eta_c \approx 10^{-6} T_6^{-3/2} \Omega m$ [Berger, 1984]. Using this result and writing $\gamma = \pi/4$; $A \approx L^2$; $L = L_R.R_e$ (1 R_E is an Earth radius, 6.4×10^6 m); $\Delta t = \Delta t_2.10^2$ s; $T = T_6.10^6$ K and $\mu_0 = 4\pi \times 10^{-7}$ we find

$$\Delta H/H < 3.4 \times 10^{-3} (\Delta t_2 \eta')^{\frac{1}{2}} / L_R$$

$$< 3.4 \times 10^{-6} (\Delta t_2 \eta' / \eta_c)^{\frac{1}{2}} / [L_R T_6^{3/4}]$$
(A5)

Clearly the ratio of the true averaged resistivity to the classical resistivity would have to be extremely large before helicity dissipation becomes significant. We can estimate the dissipation rate for dayside reconnection by choosing suitable values for Δt_2 , η' and L_R as follows: The magnetopause current sheet has a thickness (w) ~ 500 km, and since B \sim 50 nT the current density is of order $2B/(\mu_0 w) \approx 1.6 \times 10^{-7} \text{ Am}^{-2}$. The inflow speed of plasma toward the reconnection site (u) can be estimated from the duration of the reconnection event ($\Delta t \approx 3 \text{ min}$) and the diameter of the reconnected flux tube (a ≈ 1 $R_{\rm F}$). Along the reconnection line the resistivity is given by the ratio of the electric field (uB) and the current density, suggesting $\eta \approx 10^4 \Omega m$. We require the average resistivity. Since most diffusion will occur within the magnetopause current sheet, η' can be approximated by $\eta.(w/2a) \approx 390 \ \Omega m$. Using these values for L, η' and Δt in (A5) it follows that $\Delta H/H \leq 0.09$, i.e. helicity should be conserved to better than 10%. This limit on helicity dissipation is an upper limit for two reasons: The energy dissipation term due to perpendicular currents has been omitted in (A2). This has the effect of making the l.h.s. even smaller than the r.h.s. in that equation. Secondly, the estimate for the final energy (W_f) is taken as the lowest possible value (that of two untwisted flux tubes). If the two final flux tubes have some twist and a larger value of W_f is appropriate, then $\Delta H/H$ will be reduced, according to (A3). For the example discussed above (where no more than 10% of the helicity is dissipated during reconnection) the reconnected flux tubes would have some twist, suggesting our estimate of W_f is too low, and hence the upper limit on $\Delta H/H$ could be less than that given in (A5) of 10%.

Appendix B

In this appendix the partition of mutual helicity that is released during the reconnection of two flux bundles is discussed. The two flux bundles have been distorted where they cross one another, and the local picture is shown in Figure 4. Each flux bundle (Φ) has been subdivided into N flux tubes of equal flux ($\Phi_{ai}=\Phi_{bj}=\Phi/N$). The flux bundles are required to be reconnected according to the following criteria: The upper/lower incoming reconnected flux bundle maps entirely to the upper/lower outgoing flux bundle; there is no linkage of any upper reconnected field lines with any lower reconnected field lines.

The problem can be viewed as a NxN grid (N=4 in Figure 4) over the reconnection area. The flux



Fig. 4. Two flux bundles have been subdivided into N flux tubes of equal flux Φ/N . The flux bundles reconnect via reconnection of the smaller flux tubes with one another.

entering/exiting any side of the grid also maps to the same quadrant of the figure (i.e., flux entering the top left-hand side of the grid originates from the top left-hand corner of the figure). Consider the flux tube Φ_{a_1} . If reconnected to Φ_{b_1} this would satisfy our criteria. Could Φ_{a_1} reconnect to Φ_{b_2} instead? The reconnected upper flux tube $\Phi_{a_1} - \Phi_{b_2}$ satisfies the mapping condition. However, the flux tube Φ_{b_1} will violate this condition, unless it is reconnected. When Φ_{b_1} is reconnected with Φ_{ai} , the lower tube will be linked (with the upper $\Phi_{a_1} - \Phi_{b_2}$) for $i \ge 2$. (That is, if the criteria are to be met, it is necessary, but not sufficient, that reconnection occur at the $\Phi_{a_1} - \Phi_{b_1}$ crossover.)

Given that reconnection takes place at the left-hand corner grid point, we need only consider the reduced grid (N-1)x(N-1), neglecting the Φ_{a1}, Φ_{b1} grid points. (This can be done because additional reconnection between Φ_{a1} or Φ_{b1} with any Φ_{bj} or Φ_{ai} will not affect the requirement that flux entering/exiting any side of the grid maps to that quadrant of the figure.) By applying the previous analysis to the reduced grid we find that it is necessary for reconnection to occur at the $\Phi_{a2}-\Phi_{b2}$ crossover. Repeating this procedure it is evident that reconnection must take place along the horizontal axis of the diamond-shaped grid. There may be further reconnection at any other grid points too.

It is now possible to calculate the helicity of the upper and lower flux bundles after reconnection. This is done most easily by summing the helicity contributions from the flux tubes. The initial crossover of Φ_{a1} with Φ_{b_1} gave rise to a helicity of $(\Phi/N)^2$. After reconnection this will be distributed as self-helicity between the reconnected flux tubes. We shall assume that helicity is conserved during reconnection, but we shall not require that the new self-helicity be divided equally between the reconnected fluxes. The upper reconnected tube can have an increased self-helicity of $\alpha_1(\Phi/N)^2$, and the lower one $(1-\alpha_1).(\Phi/N)^2$. At crossovers off the horizontal axis where reconnection may not have occurred (e.g., $\Phi_{a_3} - \Phi_{b_2}$ in the upper flux bundle), the crossover will contribute $(\Phi/N)^2$ to the helicity of the (upper) flux bundle. If reconnection does occur at this site, then the helicity $(\Phi/N)^2$ will be divided (perhaps unequally) between the two reconnected flux tubes. Since both tubes involved belong to the same reconnected flux bundle and we are only interested in the sum of helicities for each bundle, we shall simply allow a contribution of $(\Phi/N)^2$ to the total helicity for each grid point, regardless of the reconnection details off the horizontal axis.

Thus the total helicities for the upper and lower reconnected flux bundles are

$$H_{u} = (\frac{1}{2}N^{2}-N) \cdot (\Phi/N)^{2} + \Sigma \alpha_{i} (\Phi/N)^{2}$$
(B1)

$$H_{i} = (\frac{1}{2}N^{2}-N) \cdot (\Phi/N)^{2} + \Sigma(1-\alpha_{i}) \cdot (\Phi/N)^{2}$$
(B2)

If the flux bundles are divided very finely $(N \rightarrow \infty)$, the mutual helicity of the initial flux bundles (Φ^2) is partitioned equally between the reconnected flux bundles $(H_{i\ell}, H_{\ell} \rightarrow \frac{1}{2}\Phi^2)$, under the criteria stated at the begining of this appendix.

<u>Acknowledgments</u>. The authors are grateful to N. Murphy for useful discussions. M.A.B was funded by a UK SERC research grant and A.N.W was funded by a UK SERC fellowship.

The Editor thanks two referees for their assistance in evaluating this paper.

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(Received July 26, 1988; revised October 7, 1988; accepted October 11, 1988)