THE INTERIOR STRUCTURE OF RECONNECTED FLUX TUBES IN A SHEARED PLASMA FLOW

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Two basic processes for producing twisted Abstract. magnetic flux tubes are reconnection between skewed magnetic fields, and plasma motion causing twining or braiding of flux tubes/flux elements. The properties of the latter mechanism are studied for flux that is reconnected across a neutral sheet. If the neutral sheet is formed by skewed magnetic fields with a flow shear across the sheet, the direction of the twist may be predicted in terms of background magnetoplasma quantities. If the results are applied to flux transfer events (FTEs) we expect that the vast majority of FTEs have a twist and axial By component determined by IMF By. However, we also expect there to be a few cases where the flow shear will reverse the sense of both of these quantities.

1. Introduction

Twisted and tangled magnetic fields naturally occur in many situations such as the solar corona and planetary magnetospheres. Specific features in the solar corona (e.g., a twisted flux tube, braided tubes, crossed flux tubes, arcades) have been described in terms of magnetic helicity [Berger, 1988], which is indeed a natural measure of the topology of a magnetic field [Berger and Field, 1984]. In this paper we continue to discuss the twisting and braiding of magnetic flux tubes using helicity, placing particular emphasis on twisting.

When space is divided up into distinct flux tubes, the total helicity may be split into two contributions [Berger and Field, 1984; Song and Lysak, 1989; Wright and Berger, 1989]. There is mutual helicity due to the linking and knotting of different flux tubes with one another, and there is self-helicity due to the interior structure of each flux tube (i.e., twisting and kinking). In a high magnetic Reynolds number plasma, Taylor [1974] suggested that the total helicity would be approximately conserved, even during reconnection. More recently Berger [1984] has put rigorous limits upon helicity dissipation, and it has been shown that total helicity is conserved to an excellent approximation during the reconnection of terrestrial and solar fields at the dayside magnetopause [Song and Lysak, 1989; Wright and Berger, 1989].

Satellite observations of sporadic dayside reconnection has identified plasma and field signatures, referred to as Flux Transfer Events, that are thought to be due to the motion of newly reconnected flux [Russell and Elphic,

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Paper number 89JA03517. 0148-0227/90/89JA-03517\$05.00 1978]. The interior of the reconnected flux tubes (RFTs) is found to be twisted [Cowley, 1982; Paschmann et al., 1982; Saunders et al., 1984]. The (statistical) direction of the twist can be predicted given the sense of IMF B_y and the hemisphere to which the RFT is connected [Cowley, 1982]. Over the past few years considerable effort has been devoted to investigating the mechanism which produces the observed internal twist [Lee and Fu, 1985; Sato et al., 1986; Sonnerup, 1987; Wright, 1987; Scholer, 1988a,b; Southwood et al., 1988; Song and Lysak, 1989; Wright and Berger, 1989]. The models can be classified broadly as single reconnection line models, and those employing multiple reconnection sites.

There are two fundamental processes whereby a magnetic flux tube may become internally twisted. The first one is when reconnection changes the linkage of flux tubes (i.e., the mutual helicity). If helicity is approximately conserved we shall require an equal but opposite change in the self-helicity of the reconnected flux tubes. This would most easily manifest itself as a twist. Using this mechanism the question "why are FTEs twisted?" is answered simply that helicity is not dissipated sufficiently rapidly to form untwisted RFTs. The second basic mechanism relies on some plasma motion to move the "foot points" of the flux tubes relative to one another. This may cause an individual tube to become twisted, or different flux tubes to twine about each other. In either case helicity flows across the boundary (containing the foot points) to account for changes in the helicity (i.e. topological complexity) of the magnetic field [Berger, 1988a].

We shall confine our attention to the fate of flux tubes that are reconnected across a neutral sheet. The first mechanism described above is effective in this situation when the field directions on either side of the neutral sheet are skewed relative to each other. No background plasma motion is required, and we shall refer to this mechanism as the "skewed field" mechanism, [see Wright, 1987; Scholer 1988a; Southwood et al., 1988; Song and Lysak, 1989; Wright and Berger, 1989]. If the two field directions are antiparallel, then reconnection produces two bent (but untwisted) flux tubes. Both Wright [1987] and Southwood et al. [1988] have noted that if the feet of these RFTs are moved along the neutral sheet, the flux tubes become internally twisted. Indeed this is the simplest example of the second mechanism mentioned above. For this mechanism to operate it is sufficient that there is a difference in plasma flow either side of the neutral sheet. (The relative velocity must have a component orthogonal to the two antiparallel field directions.) We shall refer to the latter mechanism as "sheared flow" [c.f. Southwood et al., 1988]. The calculations that we present here enable us to describe the twisting of magnetic flux tubes due to sheared plasma flows in a quantitative fashion, complementing earlier qualitative discussions.

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It turns out that single X-line reconnection in skewed fields and no plasma flow shear produces RFTs with the same sense of twist, when viewed along the direction of the magnetic field [Wright, 1987; Wright and Berger, 1989]. We show here that flux tubes produced by reconnection in antiparallel magnetic fields in the presence of a constant flow shear have an opposite twist to each other. For a general skewed neutral sheet with arbitrary flow shear, the two mechanisms will compete with one another. We have been able to classify (interms of background quantities) when each mechanism will be dominant, and hence which sense of twist is expected. When this criterion is applied to FTEs we expect that the vast majority of RFTs will have a twist sense governed by IMF By. However, we are able to define conditions when the plasma flow shear will reverse the direction of twist, giving an "irregular" magnetic signature.

The paper is structured as follows; section 2 derives the results needed to analyse the flow of helicity across a boundary; section 3 studies the twisting of reconnected fluxes in antiparallel fields and sheared plasma flow; section 4 extends the results of the preceeding Section by introducing skewed magnetic fields; section 5 applies the results to dayside reconnection; section 6 summarizes and concludes the paper.

2. Helicity and Helicity Flux Equations

The magnetic helicity of a subvolume of space, and the flux of helicity entering that subvolume has received much attention in the past. We shall quote useful results that are taken from Berger and Field [1984], Jensen and Chu [1984], Berger [1986, 1988a,b] and references therein. We shall assume that all magnetic field lines have both endpoints in the plane $z=z_0$, and consider the helicity of the subvolume $z>z_0$. Furthermore, this space will be divided up into portions containing magnetic flux. The total (relative) helicity may then be expressed as a sum of self- and mutual helicities.

Self-Helicity

For an individual element (i) of the magnetic field with a uniform twist T, the self-helicity may be written

$$H_i = \pm T \Phi_i^2 \tag{1}$$

T is the angle through which field lines rotate about the tube axis divided by 2π , and Φ_i is the magnetic flux of the element *i*. The flux emerges from the plane $z=z_0$ at r_{+i} and returns at r_{-i} . If the footpoints remain fixed the self-helicity of the tube may change due to rotational fluid motions centred on r_{+i} and r_{-i} (i.e. a twisting of the flux tube element about its axis r_{\pm}). If the footpoints rotate at a rate ω_{+i} and ω_{-i} respectively, then self-helicity evolves according to

$$\frac{dH_{i}}{dt} = \frac{1}{\pi} \left[\left[B_{Z}^{2}(\mathbf{r}_{-i}) \omega_{-i} d^{2} \mathbf{r}_{-i} - B_{Z}^{2}(\mathbf{r}_{+i}) \omega_{+i} d^{2} \mathbf{r}_{+i} \right] \right]$$
(2)

If the footpoints are not being turned ($\omega_{\pm i}=0$) the self-helicity may still change if the footpoint move in a simple linear fashion relative to one another. Define

$$\tan(\theta_{i}) = (\mathbf{y}_{+i} - \mathbf{y}_{-i})/(\mathbf{x}_{+i} - \mathbf{x}_{-i})$$

$$\hat{\theta}_{i} = \hat{\mathbf{z}}_{\wedge}(\mathbf{r}_{+i} - \mathbf{r}_{-i})/|\mathbf{r}_{+i} - \mathbf{r}_{-i}|$$
(3)

The angle θ_i is a measure of the linkage of the two footpoints. For example if a tube is initially untwisted and θ_i increases by 2π the new positions of the footpoints may coincide with the old ones. The new tube has two complete twists in it (T=2 in equation (1)), and the change in self-helicity is governed by the equation

$$\frac{\mathrm{d}H_i}{\mathrm{d}t} = \frac{-1}{2\pi} \left| \mathbf{B}_{\mathbf{Z}}(\mathbf{r}_{+i}) \mathbf{B}(\mathbf{r}_{-i}) \frac{\mathrm{d}\theta}{\mathrm{d}t} i \cdot d^2 \mathbf{r}_{+i} d^2 \mathbf{r}_{-i} \right|$$
(4)

Note that in this case there is no net rotation of field lines about the tube axis at a given footpoint, however the foot points themselves (at \mathbf{r}_{+i} and \mathbf{r}_{-i}) rotate about each other. Equations (2) and (4) describe the flux of magnetic helicity across the plane $z=z_{0}$, that is required to account for the changing self-helicity of the element Φ_i . If $\omega_{\pm i}$ and $d\theta_i/dt$ are both nonzero, then contributions from (2) and (4) can be added together.

Mutual Helicity

Total magnetic helicity depends upon the interior structure of each flux element (self-helicity) and the linking and knotting of different flux elements with one another (mutual helicity). Consider two flux elements Φ_i and Φ_j with footpoints $(\mathbf{r}_{+i},\mathbf{r}_{-i})$ and $(\mathbf{r}_{+j},\mathbf{r}_{-j})$ as shown in Figure 1. The two angles v and ρ are constructed by joining the footpoints of the elements *i* and *j* and labeling the angles at the feet of the tube that passes over the other flux element. If the cross section of the tubes is much less than the separation of the footpoints the mutual helicity of the two elements is given by $2H_{ij}$

$$H_{ij} = \pm \Phi_i \Phi_j (\upsilon + \rho) / 2\pi$$
(5)

The sense of H_{ij} depends upon the order in which the fluxes cross one another. In Figure 1 H_{ij} is positive. For example, if the position of the two footpoints of Φ_i were moved under Φ_i and brought very close together,



Fig. 1. The two flux tubes *i* and *j* have pairs of footpoints in the plane $z=z_0$ of $(\mathbf{r}_{+i},\mathbf{r}_{-i})$ and $(\mathbf{r}_{+j},\mathbf{r}_{-j})$ respectively. The angles v and ρ are formed by constructing lines between the footpoints. The mutual helicity of this configuration is $2H_{ij}=\Phi_i\Phi_j(v+\rho)/\pi$.

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then $\nu + \rho \approx 2\pi$ and Φ_i would link with Φ_j once. In this case equation (5) yields the familiar result $2H_{ij} = +2\Phi_i \Phi_j$. For tubes with arbitary cross section and footpoint locations, the mutual helicity $(2H_{ij})$ may be found more generally from

$$H_{ij} = \frac{+1}{2\pi} \left[B_{z}(r_{+i}) B_{z}(r_{+j}) (\nu + \rho) d^{2}r_{+i} d^{2}r_{+j} + n \Phi_{i} \Phi_{j}(6) \right]_{z=z_{0}}$$

where v and ρ are functions of \mathbf{r}_{+i} and \mathbf{r}_{+j} . The last term takes into account that Φ_i and Φ_j may also twine about one another an integer number of times (n).

The rate of change of mutual helicity can be found from the derivative of equation (6) in terms of $d\nu/dt$ and $d\rho/dt$. An equivalent expression that is sometimes used is

$$\frac{\mathrm{dH}}{\mathrm{dt}}ij = \frac{-1}{2\pi} \left[\mathbf{B}_{z}(\mathbf{r}_{a})\mathbf{B}_{z}(\mathbf{r}_{b}) \cdot \frac{\mathrm{d\theta}}{\mathrm{dt}}ab \cdot d^{2}\mathbf{r}_{a}d^{2}\mathbf{r}_{b} \right]$$
(7)

The right hand side is summed over the pairs (a,b) = (+i,+j), (+i,-j), (-i,+j), (-i,-j).

$$\tan(\theta_{ab}) = (y_b - y_a)/(x_b - x_a)$$

$$\theta_{ab} = \hat{z}_{\Lambda}(r_b - r_a)/|r_b - r_a|$$
(8)

If the footpoints of Φ_i in Figure 1 are brought together, but around the side of Φ_j rather than underneath it, the mutual helicity $2H_{ij}=0$ rather than $+2\Phi_i\Phi_j$. This is due to the differing time histories of v and ρ (or θ_{ab}) in the two cases.

Total Helicity

The total helicity of a collection of flux tubes may be calculated by suming the contributions to self- and mutual helicities,

3. Reconnected Fluxes in Antiparallel Fields and Sheared Flow

The relations for helicity and helicity flux in the last section will now be applied to magnetic flux that is reconnected across a tangential discontinuity. The unperturbed magnetoplasma (which is taken to be ideal) will be modeled in the following form

$$\mathbf{B} = \begin{cases} \mathbf{B}_{0} \hat{\mathbf{z}} & x < 0 \\ -\mathbf{B}_{0} \hat{\mathbf{z}} & x > 0 \\ & x > 0 \end{cases}$$

$$\mathbf{V} = \begin{cases} \mathbf{V} \hat{\mathbf{y}} & x < 0 \\ -\mathbf{V} \hat{\mathbf{y}} & x > 0 \end{cases}$$
(10)

Figure 2a shows two columns of flux that will be reconnected by a reconnection line of length L_y at the



Fig. 2. Two columns of flux extend between z=0 and $z=-L_z$ (a). (The field and flow are defined in equation (10).) In Figure 2b the fluxes have reconnected and their footpoints have drifted apart with the background plasma motion. In Figure 2c the fluxes did not reconnect, but are distorted to overlap with one another.

center of the box. This reconnection line could a short section of a much longer reconnection line, in which case the cross section of the flux columns will be a rhombus. If L_y represents the entire length of the reconnection line then the cross section will be slightly distorted from a

rhombus. Nevertheless, it will preserve the important property that field lines closest to the neutral sheet are reconnected at earlier times than those furthest from the neutral sheet. For the purposes of this paper we shall only consider the rhombus cross section. The reconnection rate is uniform for times 0 < t < T, and zero otherwise. The depth of the reconnected flux columns, L_{χ} , defines the total flux reconnected by the length of reconnection line L_{χ} ,

$$\Phi_0 = L_X L_V B_0 \tag{11}$$

Figure 2a shows the situation at t = 0, when the two sheets of flux with endpoints $0 \le \pm x \le \pm \Delta x$ and $-\frac{1}{2}L_y \le y \le \pm \frac{1}{2}L_y$ are reconnected. At time goes by sheets of flux further from the neutral sheet are reconnected, until at time T the last sheet with foot points $\pm L_x \le \pm x \le \pm L_x \pm \Delta x$ and $-\frac{1}{2}L_y \le y \le \pm \frac{1}{2}L_y$ is reconnected. (The rhomboid is a rectangle that has been sheared by an angle $\arctan(VT/L_x)$; thus the footpoints of the last reconnected field lines have drifted by VT which brings them into alignment with the reconnection line at time T.) After time T no more reconnection takes place and the footpoints of the columns continue to move apart, as shown in Figure 2b. By calculating the helicity of sheets of reconnected flux it is possible to evaluate the helicity of the large flux columns Φ_0 .

Flux Sheet Helicity

We shall consider the reconnection of the two columns in Figure 2a as a continuous reconnection of fine sheets of flux. In this subsection we shall confine our attention to flux sheets connected to the $z=-L_z$ plane. The *i*th flux sheet is reconnected at a time t_i and has footpoints mapping to x_i in the interval Δx at $x=\pm L_x t_i t_i^T$, $-\frac{1}{2}L_y < y < +\frac{1}{2}L_y$. The flux of the sheet is $d\Phi=\mp B_0 L_y \Delta x$. The coordinates of the endpoints of individual field lines in the *i*th flux sheet may be written (when $t > t_i$)

$$\mathbf{r}_{+i} = (-\mathbf{L}_{\mathbf{X}}\mathbf{t}_{i}/\mathbf{T}, \quad \mathbf{V}(\mathbf{t}-\mathbf{t}_{i})+1, \quad -\mathbf{L}_{\mathbf{Z}})$$

$$\mathbf{r}_{-i} = (+\mathbf{L}_{\mathbf{X}}\mathbf{t}_{i}/\mathbf{T}, \quad -\mathbf{V}(\mathbf{t}-\mathbf{t}_{i})+1, \quad -\mathbf{L}_{\mathbf{Z}})$$
(12)

The parameter 1 ranges from $-\frac{1}{2}L_y \rightarrow +\frac{1}{2}L_y$. Equation (3) yields the angle between pairs of endpoints, which turns out to be independent of 1;

$$\theta_{i} = \arctan[VT(1-t/t_{i})/L_{x}]$$
(13)

There is no rotational motion around the footpoints, so the self-helicity of this flux sheet evolves according to equation (4) which may be integrated to give

$$dH_{i}/dt = d\theta_{i}/dt . (d\Phi)^{2}/\pi$$
or
$$H_{i}(t) = \arctan[VT(1-t/t_{i})/L_{X}] . (d\Phi)^{2}/\pi$$
(14)

The second relation in equation (14) is true if $H_i(t=t_i)=0$ (i.e., the *i*th flux sheet is untwisted immediately after reconnection). H_i has an asymptotic value of $-\frac{1}{2}(d\Phi)^2$ as $t\to\infty$. This would correspond to the flux sheet evolving a negative half twist $(T=\frac{1}{2}$ in equation (1)). $H_i(t)$ is sensitive to the time of reconnection, t_i . Figure 3 shows the variation of self-helicity for three flux sheets reconnected at times $t_1=T/4$, $t_2=T/2$ and $t_3=T$. The earlier a flux sheet is reconnected the more twisted it is at later times. This is because they have longer to acquire twist, and also $d\theta_i/dt$ is greater for small t_i . The flux sheets connected to the plane z=0 have an equal, but opposite, self-helicity to their counterparts connected to $z=-L_z$.

Flux Tube Helicity

The helicity of the flux columns Φ_0 may be calculated by suming the self- and mutual helicities of their constituent fluxsheets, as in equation (9). Perhaps the easiest way to envisage these nested flux sheets (like the layers of an onion) is by considering the last sheet, or layer, to be reconnected. In Figure 2b (for the lower reconnected column) this layer has its footpoints along the front edge (at $x=+L_x$) and along the back edge (at $x=-L_x$) in the $z=-L_z$ plane. Indeed the lines shown in Figure 2b from the front corners at $x=+L_x$ are coincident



Fig. 3. The variation of self-helicity with time for three flux sheets reconnected at different times. The last sheet to be reconnected (at $t_3=T$) is slow to increase its self-helicity. The sheets reconnected earlier (at $t_2=T/2$ and $t_1=T/4$) acquire helicity much more rapidly, suggesting that the earlier a sheet is reconnected the more twisted it appears at later times.

with the field lines at the edges of the last flux sheet to be reconnected. Flux sheets that were reconnected before this outer-most flux sheet form nested surfaces. Consider the mutual helicity of two nested flux sheets Φ_i and Φ_j when t>T. Although equation (6) can be evaluated as a function of time for two flux sheets, the expression is extremely cumbersome. Moreover, the mutual helicity cannot be sumed easily over all pairs ij, so we shall not discuss $H_{ij}(t)$ further here. The behaviour of H_{ij} can be illustrated by using the following simplifying assumptions. First, we allow reconnection to occur so rapidly that the plasma motion during 0<t<T is negligible compared with L_x . Hence $\epsilon_1 = VT/L_x \leq 1$, and the cross section of the flux columns becomes rectangular. Second, we shall only consider the mutual helicity at t=T and very large t (when $\epsilon_2 = Vt/L_X \ge 1$). At t=T we have essentially two rectangular reconnected flux columns. Equation (13) gives $\theta_i=0$, and equation (14) states that $H_i=0$ (to zeroth order in ϵ_1). Also, in equation (6) $v+\rho=0+O(\epsilon_1)$, so $H_{ij}=0+O(\epsilon_1)$. Summing the self- and mutual helicities according to equation (9) one finds that the total helicity of the lower (and upper) reconnected flux column is zero to lowest order in ϵ_1 .

At much later times $(\epsilon_2 \text{ large})$ the self-helicity of each flux sheet tends to $-\frac{1}{2}(d\Phi_i)^2$ according to equation (14), for endpoints at $z=-L_z$. The mutual helicity (equation (6)) is particularly simple in this limit also since $\nu+\rho=\pi+O(\epsilon_2^{-1})$ for every pair of field lines in the sheets (Φ_i, Φ_j) . Thus the mutual helicity, to lowest order in ϵ_2^{-1} , is $2H_{ij}=-\Phi_i\Phi_j$. If Φ_0 is divided into N equal sheets of flux then $d\Phi=\Phi_i=\Phi_0/N$, and the total helicity of the lower flux column (equation (9)) becomes

$$H_{l} = -N \cdot \frac{1}{2} (\Phi_{0}/N)^{2} - \frac{1}{2} (N^{2}-N) \cdot (\Phi_{0}/N)^{2}$$

$$= -\frac{1}{2} \Phi_{0}^{2}$$
(15)

This would correspond to the lower flux column acquiring a negative half twist (to lowest order in ϵ_1 and $1/\epsilon_2$). If a similar calculation is done for the upper reconnected flux column, all the helicities have the opposite sense. Hence the upper flux column (in Figure 2b) has a positive half twist.

It is interesting to note that the helicity inside the box shown in Figure 2 due to upper and lower reconnected fluxes is always zero. This is because the helicity flowing into the upper flux elements (across the plane z=0) is equal to the helicity flow out of the lower flux elements (across the plane $z=-L_z$). One may also intuit this result with the following argument, given that helicity dissipation is neglected during reconnection: Consider the two flux columns in Figure 2a. If no reconnection occurs then the two columns will simply move apart along the y axis. In Figure 2c the footpoints have drifted to the same locations as the reconnected columns in Figure 2b, and the tubes have been bent so that they overlap each other. Distorting the tubes does not change any of the helicities. Each tube is still untwisted, however the two cross overs contribute to the mutual helicity of the flux columns. The upper/lower cross over is positive/negative and produces a mutual helicity of $+\Phi_0^2/-\Phi_0^2$. Thus the total mutual is still zero. The two flux columns can now be reconnected via localized reconnection at one of the cross overs. It does not matter which site reconnection takes place at, so we shall choose the upper, positive crossover. Wright and Berger [1989] have shown that when two crossed flux tubes are reconnected the decrease in mutual helicity (Φ_0^2) is divided equally between the reconnected flux tubes and increases their self-helicity. Hence reconnection at the upper crossover in Figure 2c will produce two reconnected flux tubes, and the upper one will have a self-helicity of $\frac{1}{2}\Phi_0^2$. The lower reconnected tube has a contribution of $\frac{1}{2}\Phi_0^2$ to its self-helicity from reconnection at the upper crossover, and $-\Phi_0^2$ due to the negative crossover with itself (i.e., a net self-helicity of $-\frac{1}{2}\Phi_0^2$). Both reconnected flux columns in Figure 2c may relax to a shape like that shown in Figure 2b, and by the above analysis we expect the upper one to have a positive half twist, and the lower one a negative half twist in agreement with the earlier calculation. This suggests that the corrections of order ϵ_1 and $1/\epsilon_2$ are identically zero.

4. Reconnected Fluxes in Skewed Fields and Sheared Flows

When reconnection occurs across a neutral sheet where there are skewed magnetic fields and no background plasma motion, both the reconnected flux tubes (RFTs) gain a half twist. The sense of the twist is the same for both tubes and is determined by the sense of the crossover prior to reconnection [Wright and Berger, 1989]. The last section showed that reconnected fluxes in antiparallel magnetic fields and sheared flows produces two RFTs that both have a half twist, but always of opposite senses. In this section we consider reconnection across a general neutral sheet that has skewed magnetic fields and sheared flows. Both the "skewed field" and "sheared flow" mechanisms will compete to twist the RFTs, and a full calculation of this problem is beyond the scope of this paper. However, we give below an approximate criterion for deciding which mechanism will dominate.

Figure 4a shows an element of flux immediately after having been reconnected between the fields B₁ and B₂ which form a tangential discontinuity. The motion of the reconnected tube is complex, particularly for sporadic reconnection. The plasma motion is communicated from the reconnection site (r) along the RFT by Alfvén waves. Figure 4a only shows half of the reconnected flux. This half propagates Alfvén waves at speeds V_{A_1} and $-V_{A_2}$ along B_1 and B_2 , respectively. (The lower tube that is not shown would excite waves with velocities $-V_{A_1}$ and V_{A_2} .) After a time t has elapsed from reconnection information will have reached the points p and q on the RFT (assuming no background plasma motion, V_{21}). It is not possible to describe the magnetic structure in detail, however we do know that beyond the points p and q there are no MHD disturbances and also that the field lines pass from p to q. The tube shown in Figure 4b is the simplest configuration, althought it is possible that the section between p and q could be bent [cf. Sonnerup, 1987].

If there is some relative plasma motion either side of the neutral sheet $(V_{21}\neq 0)$ we may get configurations like those in Figures 4c and 4d. The figures are drawn in the rest frame of the plasma on B_1 , so q remains unchanged but p is displaced by $V_{21}t$. In Figures 4b and 4c the "skewed field" mehanism dominates and the section of tube between p and q has a positive half twist. In Figure 4d the "sheared flow" mechanism dominates and reverses the twist of the tube between points p and q to a negative half twist. It is possible to define which mechanism will dominate in terms of the quantities V_{A1} , V_{A2} and V_{21} . Define the velocity U

$$\mathbf{U} = \mathbf{V}_{A_1} + \mathbf{V}_{A_2} \neq \mathbf{V}_{21} \tag{16}$$

The upper and lower signs correspond to the upper and lower reconnected fluxes. In Figure 4 the direction of the



Fig. 4. A neutral sheet exists at the boundary of two magnetic fields \mathbf{B}_1 and \mathbf{B}_2 . There is some rapid localized reconnection at the site r between the fields \mathbf{B}_1 and \mathbf{B}_2 : (a) A reconnected field line is shown immediately after reconnection. (b) If there is no relative plasma motion, information is communicated along the field lines from the reconnection site, r, at the local Alfvén speeds to sites p and q. (c) Relative plasma motion across the neutral sheet may distort the tube without altering its internal twist. (d) In some cases the plasma motion may reverse the sense of twist of the flux tube.



Fig. 5. Reconnection at two long parallel reconnection lines (shown in bold). The flux rope formed between the reconnection lines can be characterized by its sense of twist and axial field direction. If field lines are reconnected at both lines at the same time, then no matter how large V_{21} is, the sense of twist and axial field are unchanged. However, if condition (17b) is satisfied, these quantities may be reversed provided that following reconnection at site b, point a has time to drift beyond point c before being reconnected by the upper reconnection line.

magnetic field lines joining p and q is coincident with the direction of U. If $\varphi(C)$ is the angle of any vector C projected onto the neutral sheet then the "skewed field" mechanism dominates when

$$\varphi(-\mathbf{V}_{\mathbf{A}_1}) < \varphi(\mathbf{U}) < \varphi(-\mathbf{V}_{\mathbf{A}_2}) \tag{17a}$$

and the "sheared flow" mechanism dominates when

$$\varphi(-\mathbf{V}_{\mathbf{A}_2}) < \varphi(\mathbf{U}) < \varphi(-\mathbf{V}_{\mathbf{A}_1}) \tag{17b}$$

Note that this criterion is only approximate because we have assumed that the Alfvén waves launched from r are quite localized. The discussion above can be applied to sporadic three-dimensional reconnection, but is rather heuristic. Steady reconnection along an infinite reconnection line has been modeled quantitatively by Biernat et al. [1988], who also found that inside the reconnection layer the magnetic field was aligned with U. This agrees with our qualitative discussion, and the criteria (17) may be used when considering steady reconnection as discussed by Biernat et al. [1988], in addition to localized sporadic reconnection.

5. Discussion

An obvious application of the results derived here is dayside reconnection between the terrestrial and solar wind magnetic fields. In this case Figure 4 would

correspond to the northern connected flux tube viewed toward the Earth with IMF $B_z < 0$ and IMF $B_y > 0$ (B_1 is the magnetospheric field and B_2 is the magnetosheath field). Typical dayside parameters are $V_{A_1} \sim 600$ km s⁻¹, $V_{A_2} \sim V_{21} \sim 250$ km s⁻¹. This ordering of quantities means that it will not be possible for the "sheared flow" mechanism (equation (17b)) to dominate when there is a significant IMF B_y component. Thus we anticipate that the vast majority of FTE observations will have a twist governed by the "skewed field" mechanism [Wright and Berger, 1989], which is sensitive to the sign of IMF By as reported by Cowley [1982] and Paschmann et al. [1982]. The skewed field mechanism also has the property that the By component inside the reconnection layer (between p and q in Figures 4b and 4c) is in the same direction as the IMF By component.

The sheared flow mechanism is likely to come into operation during dayside reconnection when there is a strong magnetosheath flow in the IMF 9 direction and the fields are approximately antiparallel. In this case (if equation (17b) is valid) the twist sense will be opposite to that predicted by Cowley [1982] and Paschmann et al. [1982]. If a spacecraft were to enter such a RFT, the B_n signature would be discontinuous, and the sense of B_v between p and q in Figure 4d would be opposite to that of IMF B_v. It would be interesting to look for such anomalous signatures in the data.

Besides single reconnection line models of FTEs there are multiple reconnection line models [e.g., Lee and Fu, 1985; Fu and Lee, 1986]. The role of a flow shear in determining the sense of twist depends upon the sequence in which a given field line is reconnected at the different neutral lines. For exmaple, if the field line B, in Figure 5 is reconnected at the same instant at both reconnection lines then the flow shear has no effect upon the magnetic geometry of the flux rope produced between the two reconnection lines. On the other hand, if the field line is reconnected at the lower line first (site b), then the new field line may evolve like those in Figure 4. If it evolves in a similar fashion to that depicted in Figures 4b or 4c, then no reverse twist can be produced. However, if condition (17b) is valid and the field line assumes a shape like that in Figure 4d it is possible for the twist sense to reverse if reconnection at the upper line occurs after point a has drifted beyond point c. When this is satisfied the sense of twist and axial field of the flux

rope will be opposite to that if $V_{2,1}=0$. The remaining models of FTEs are due to Scholer [1988a,b] and Southwood et al., [1988] who both discuss the bubble-like structures produced by unsteady reconnection along a single long reconnection line. It is interesting to compare these models with the single short reconnection line model of Russell and Elphic [1978] (see also Wright [1987]). The velocity U in equation (16) corresponds to the speed at which two Alfvén waves on either side of the neutral sheet separate. This can be combined with the length of the reconnection line, L_y , to the abarecteristic time $\tau = L_z/U$. If the RFT is give a characteristic time, $\tau = L_v/U$. If the RFT observed a time t after reconnection, and $t < \tau$, then the quasi two-dimensional models of Scholer [1988a] and Southwood et al. [1988] will be appropriate. If the time of observation after reconnection is greater than τ , the three-dimensional nature of the RFT becomes important and the isolated flux tubes suggested by Russell and Elphic [1978] may be a more useful way to describe FTEs. Clearly the length of the reconnection line is an important parameter. Scholer [1988b] has estimated this length to be as much as 5R_E. (His calculation requires that an equilibrium is achieved between the poleward force of the RFT and the equatorward force of the "magnetopause field". This may not be realized -

especially if the magnetopause field is bent less than Scholer suggests.)

6. Conclusions

There are two principal methods for producing twisted magnetic flux tubes within MHD theory. One relies upon reconnection redistributing helicity amongst the RFTs in such a way that their internal structure changes, i.e., they gain a positive or negative twist. This is most easily achieved during reconnection between magnetic fields that are skewed relative to one another [Wright and Berger, 1989]. The other method requires some sort of fluid motion to braid or twine the magnetic field lines about each other [Berger, 1988b]. We have concentrated upon the latter mechanism and studied its effect on RFTs across a tangential discontinuity. We find that a pair of RFTs in antiparallel fields and a flow shear will steadily increase their twist up to a maximum of a half twist. The sense of twisting in a given flux tube is always opposite to that its partner. It can also be shown that the elements of flux reconnected first become more twisted than those reconnected later [cf. Southwood et al., 1988].

When the neutral sheet is formed by magnetic fields that are skewed relative to each other and has no flow shear, the pair of RFTs both have the same sense of twist. If a flow shear is introduced both twisting mechanisms will compete with one another. An approximate relation is derived (equation (17)) for determining which twisting mechanism will dominate. When this condition is applied to dayside reconnection, we find that the skewed field mechanism will nearly always dominate. Nevertheless there will be some occasions when the sheared flow mechanism predominates and this is expected to produce FTEs with an irregular twist and axial By component.

Acknowledgments. This work was carried out while both authors were U.K. SERC research assistants.

The Editor thanks M. Hesse and another referee for their assistance in evaluating this paper.

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> (Received June 27, 1989; revised October 11, 1989; accepted November 6, 1989.)