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Field-aligned electron acceleration in Alfvén waves

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[1] The field-aligned current of standing Alfvén waves is mainly carried by electrons travelling parallel to the magnetic field. During the upward current phase, magnetospheric electrons travel downward to the ionosphere. In large-amplitude Alfvén waves, where current densities reach a few μAm^{-2} above the ionosphere, the electrons achieve energies of the order of keV. This problem has been addressed recently in terms of two-fluid theory. The present paper builds on these studies by employing a distribution function formulation. When the electron motion is dominated by the parallel velocity component, we find the B/n curve is central to interpreting the solution: B/n has a peak (i.e., $d(B/n)/d\ell = 0$, where ℓ is path length along the field line) below which ionospheric electrons are trapped. Above the peak we find the parallel electric field is balanced by the convective plasma acceleration, as suggested by *Rönnmark* [1999] and has a value of the order of mV/m for $\sim 1\ R_E$ above the B/n peak. The maximum $E_{\|}$ occurs where $d^2(B^2/n^2)/d\ell^2 = 0$ and is located a couple of density scale heights beyond the B/n peak. *INDEX TERMS:* 2451 Ionosphere: Particle acceleration; 2407 Ionosphere: Auroral ionosphere (2704); 2409 Ionosphere: Current systems (2708); 2431 Ionosphere: Ionosphere/magnetosphere interactions (2736); *KEYWORDS:* electron acceleration, Alfvén wave, aurora, parallel electric field, Vlasov equation

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1. Introduction

[2] The single-fluid Magnetohydrodynamic (MHD) approximation has proved to be very successful for describing ULF waves in the Earth's magnetosphere: *Southwood* [1974], *Chen and Hasegawa* [1974], and *Wright and Thompson* [1994] showed how resonant Alfvén waves (referred to as field line resonances, or FLRs) could be excited preferentially on certain L-shells.

[3] Numerical simulations demonstrated that fast cavity modes [*Allan et al.*, 1985; *Lee and Lysak*, 1989] or waveguide modes [*Wright*, 1994; *Rickard and Wright*, 1994] were the likely agent for exciting Ultra-Low-Frequency (ULF) Alfvén waves, whose existence is very well established observationally [e.g., *Samson et al.*, 1971; *Walker et al.*, 1979]. Alfvén waves are well known for the field-aligned current they carry, and the last decade has established a strong link in some observations between optical auroral emissions and Alfvén wave fields [*Samson et al.*, 1991, 1992; *Xu et al.*, 1993; *Liu et al.*, 1995]. These studies have demonstrated that the auroral luminosity is modulated with the same frequency as simultaneously observed Alfvén wave fields. This suggests the periodic precipitation of energetic field-aligned electrons into the ionosphere, associated with the field-aligned current, are modulating the optical emissions.

[4] The energy of the precipitating particles for a current of $\sim\mu A/m^2$ is $\sim$keV, and interest has focused recently on how the electrons are accelerated. The traditional, and very successful, single-fluid MHD approximation is not suitable for this work: formally this limit neglects the electron mass compared to that of the ions. Massless electrons are infinitely mobile, and can move with ease to where they are required to preserve charge neutrality and carry the required current ($\nabla_\wedge \mathbf{B}/\mu_0$). Electrons of negligible mass require a negligible electric field to accelerate them to high speeds and corresponding negligible energies. Evidently this limit is of little use for investigating the electric fields thought to be responsible for accelerating electrons to energies of the order of a keV.

[5] Early studies of $E_{\|}$ generation were based upon the requirement of quasi-neutrality in a mirroring distribution of electrons and ions [*Alfvén and Fälthammar*, 1963; *Persson*, 1966]. Subsequently $E_{\|}$ associated with the equilibrium of different plasmas, such as a hot diffuse magnetospheric plasma bounded by a cold dense ionospheric plasma were considered [*Chiu and Schultz*, 1978; *Stern*, 1981]. These studies found potential differences along auroral field lines of the order of a kilo-Volt. *Chiu and Schultz* [1978] found the $E_{\|}$ was concentrated between altitudes of 1 to 2 R_E and had a value of 0.5 mV/m, whereas *Stern*'s [1981] solutions favored the formation of double layers.

[6] The importance of electron inertia in auroral currents was first stressed by *Goertz and Boswell* [1979]. Recently, studies have retained a finite electron mass and used the two-fluid equations (one fluid for the electrons, and another for the ions) to study FLRs. *Streltsov and Lotko* [1997] and *Streltsov et al.* [1998] showed how finite electron inertia introduces dispersion and can produce latitudinal structuring [see also *Liu et al.*, 1995]. *Rönnmark* [1999] suggested

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that the large observed electron speeds and finite electron mass could account for parallel electric fields of the order of mV/m at altitudes of $\sim$1 R_E. *Wright et al.* [2002] applied Rönnmark's ideas to FLRs and found similar energization.

[7] Although the two-fluid equations permit an investigation of electron energization, this approximation only yields information about the center of mass speed of the distribution, and so fails to capture a lot of the details of the complete solution, such as the penetration of an energetic beam of magnetospheric electrons into a dense low energy ionospheric plasma. *Rönnmark and Hamrin* [2000] addressed this problem within the electron fluid approximation by using separate fluids for the magnetospheric and ionospheric populations. *Wright et al.* [2002] identified the regions of the FLR flux tube where a single electron fluid would be useful, and discussed the qualitative form of the solution elsewhere.

[8] A more accurate scheme for describing the electrons is to use an electron distribution function, and is the approach we adopt in this article. Following observations that show the accelerated electrons have parallel speeds much greater than their perpendicular speed we focus on field-aligned motion as a first approximation. The limit of small perpendicular speed means magnetic mirroring is neglected. However, since this effect is not properly accounted for in studies using the electron fluid description, our calculation can be thought of as a useful refinement to these studies. We find great simplification by considering field-aligned motion to the extent that our results are analytical and provide considerable insight into the physics involved in electron acceleration. These results provide a useful benchmark with which to compare calculations which do include magnetic mirroring. For example, *Rönnmark* [2002] finds similar behavior, except that $E_\parallel$ increases by a factor of 4. *Rankin et al.* [1999] also found that mirroring could enhance $E_\parallel$, as did *Nakamura* [2000] who gives a lucid description of the physics involved.

[9] As noted by *Rönnmark* [1999] and *Wright et al.* [2002], the presence of a nonuniform (dipolar-like) magnetic field causes the field-aligned current density to increase dramatically over a scale $\sim$1 R_E. The resulting FLR electron dynamics are nonlinear, $(V_\parallel \nabla_\parallel)V_\parallel \gg \partial V_\parallel/\partial t$ ($V_\parallel$ is the parallel component of the electron fluid velocity), at altitudes of less than a few R_E for field-aligned currents of $\sim$ μA/m^2 reaching the ionosphere, even if $V_\parallel/V_A \ll 1$ (V_A being the Alfvén speed). The nonuniform magnetic field is an essential ingredient, and the increase in number density (n) in the ionosphere is a new feature that we model quantitatively in the present calculation and is possible through the use of a kinetic description.

[10] The importance of the variation of B/n has been raised in previous studies: *Swift* [1975] identified the ratio as being proportional to the mean, or fluid, electron speed. *Lysak and Hudson* [1979] found B/n peaked at altitudes of $\sim$1 R_E. Our calculation also stresses the importance of B/n. We find, in accord with previous studies, that maximum energization occurs where $d(B/n)/d\ell = 0$ (ℓ being the field-aligned coordinate). Moreover, we find the maximum $E_\parallel$ for downgoing electrons occurs at slightly higher altitudes and is identified by $d^2(B^2/n^2)/d\ell^2 = 0$. These locations are consistent with those of the field-aligned potential drop inferred by *Shiokawa et al.* [2000] based upon mapping observed electron distributions between high and low altitude satellites.

[11] The paper is structured as follows: Section 2 outlines our model and approximations. Section 3 derives the solution for the case of an upward field-aligned current. Section 4 interprets the details of our solution and compares with previous studies, and section 5 summarizes our results.

2. Model

2.1. Equilibrium

[12] The acceleration of electrons occurs at altitudes of $\sim$1 R_E and over a distance along the field line of the order of an Earth radius. Thus, we shall focus upon the section of field line within a few R_E of the Earth. The transit time of a keV electron over 1 R_E is $\sim$1 s. Given the period of a ULF wave is hundreds of seconds, we shall model the upward current phase as a steady solution by setting $\partial/\partial t = 0$. We shall show later that the neglect of $\partial/\partial t$ is consistent with the ordering $(V_\parallel \nabla_\parallel)V_\parallel \gg \partial V_\parallel/\partial t$ mentioned above and is discussed more fully in subsection 4.4. In the region of interest the field lines are approximately dipolar, and we take

$$B = B_0 \frac{\sqrt{1 + 3\sin^2\theta}}{\cos^6\theta} \tag{1}$$

where B_0 is the (notional) equatorial field strength and B is parameterized in terms of θ, the latitude. The radial distance to a point on the field line is

$$r = LR_E \cos^2\theta \tag{2}$$

and we set the L-shell to be $L = 10$, which gives field lines entering the ionosphere at a latitude of 71.5°.

[13] The ion number density (n) in the magnetosphere is taken to be constant (n_0) and in the ionosphere is stratified by gravity with a scale height h,

$$n = n_0 + (n_m - n_0)\exp(-(r - R_E)/h) \tag{3}$$

n_m is the maximum ion density at the base of the F region, which we take to be at $r = 1$ R_E for convenience. We assume the ions are singly charged and the plasma quasi-neutral so (equation (3)) is also the total number density of electrons.

[14] For illustrative purposes we take $B_0 = 25$ nT (giving B in the ionosphere of 5×10^4 nT), $n_0 = 10^6$ m^{-3}, $n_m/n_0 = 10^3$, and $h = 400$ km, and show the variation of B/n (normalized by B_0/n_0) in Figure 1. The coordinate s is the path length along the field line measured from the ionospheric end.

2.2. Electron Distribution Function

[15] We assume the plasma is collisionless in the region of interest and describe the electrons via a distribution function $f(\ell, v_\parallel, v_\perp, t)$ which depends upon the distance along the field line (ℓ), the parallel and perpendicular components of the guiding center drift velocity ($v_\parallel$ and $v_\perp$, respectively), and time (t). The gyrotropic distribution function satisfies Vlasov's equation

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial \ell} + \frac{dv_\parallel}{dt} \cdot \frac{\partial f}{\partial v_\parallel} + \frac{dv_\perp}{dt} \cdot \frac{\partial f}{\partial v_\perp} = 0 \tag{4}$$

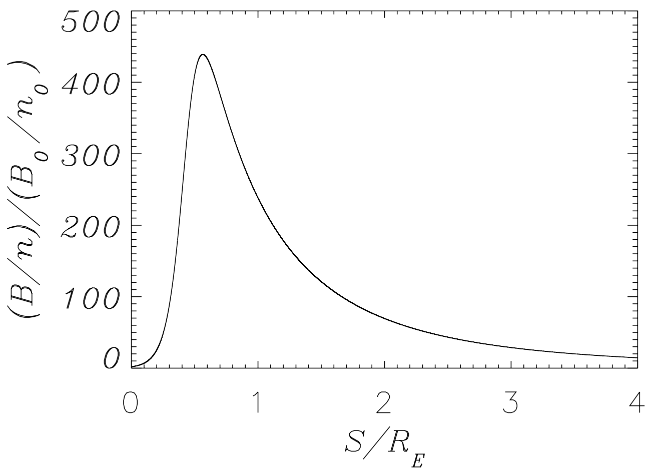

Figure 1. The variation of B/n, normalized by B_0/n_0, along an $L = 10$ field line. The path length is measured from the base of the F region ($s = \ell_m - \ell$). The peak of B/n occurs at $s/R_E = 0.56$.

and remains constant on an electron trajectory. We assume the magnetic moment

$$\mu = \frac{mv_\perp^2}{2B} \tag{5}$$

is conserved. Seeking a solution of the form

$$f = F(\ell, v_\parallel, v_\perp, t) g(\mu) \tag{6}$$

and noting that the parallel component of the electron guiding center drift velocity evolves according to $mdv_\parallel/dt = -eE_\parallel - \mu\partial B/\partial\ell$, equation (4) becomes

$$g(\mu)\left[\frac{\partial F}{\partial t} + v_\parallel \frac{\partial F}{\partial \ell} - \left(\frac{eE_\parallel(\ell, t)}{m} + \frac{v_\perp^2}{2B}\frac{dB}{d\ell}\right)\cdot\frac{\partial F}{\partial v_\parallel} + \frac{v_\parallel v_\perp}{2B}\cdot\frac{dB}{d\ell}\cdot\frac{\partial F}{\partial v_\perp}\right] = 0 \tag{7}$$

To focus on field-aligned motion we let $g(\mu) = (m/\pi)\delta(\mu)$ and integrate equation (7) over $v_\perp$ space, giving

$$B\frac{\partial F}{\partial t} + v_\parallel B \frac{\partial F}{\partial \ell} - \frac{eE_\parallel B}{m}\cdot\frac{\partial F}{\partial v_\parallel} = 0 \tag{8}$$

and $F = F(\ell, v_\parallel, v_\perp = 0, t)$. Once F has been found from equation (8) the number density (n) and field-aligned current (j) are given by

$$\frac{n(\ell, t)}{B(\ell)} = \int_{-\infty}^{+\infty} F(\ell, v_\parallel, t) dv_\parallel \tag{9a}$$

$$\frac{j(\ell, t)}{B(\ell)} = -e\int_{-\infty}^{+\infty} v_\parallel F(\ell, v_\parallel, t) dv_\parallel \tag{9b}$$

Quasi-neutrality is imposed in our calculations by requiring that the electron number density in equation (9a) be equal to the ion number density defined in equation (3). Following the suggestion that the fields are treated as steady in the acceleration region we set $\partial/\partial t = 0$ and use a potential (ϕ) to describe the electric field

$$\mathbf{E} = -\nabla\phi; \quad E = -\frac{\partial\phi}{\partial\ell} \tag{10}$$

Defining $\Phi = e\phi/m$, equation (8) reduces to

$$v_\parallel \frac{\partial F}{\partial \ell} + \frac{\partial \Phi}{\partial \ell}\cdot\frac{\partial F}{\partial v_\parallel} = 0 \tag{11}$$

which expresses the fact that the total energy of an electron (W) is conserved along a trajectory $W(\ell, v_\parallel) = \text{const.}$

$$F(W) = F(W(\ell, v_\parallel)); \qquad W = m\left(\frac{1}{2}v_\parallel^2 - \Phi(\ell)\right) \tag{12}$$

Our task is to determine the function $\Phi(\ell)$ which yields an F whose moments (equations (9a) and (9b)) give the required electron number density and current density subject to appropriate boundary conditions.

3. Upward Current Solution

3.1. Overview

[16] We begin with an outline of the results to help orientate the reader through the following calculation. Figure 2 shows a sketch of a converging flux tube parameterized in terms of the field-aligned path length (ℓ) which increases as the ionosphere is approached. In this calculation we consider upward currents, i.e., electrons are accelerated downward into the ionosphere. We shall show that the point ℓ_c is a critical position: It coincides with the peak of $B(\ell)/n(\ell)$. We take ℓ_0 to be a reference point at higher altitudes than the acceleration region, and whose exact location is not important. For convenience we let ℓ_0 be located in the equatorial plane.

[17] Over the section $\ell_0 < \ell < \ell_c$ we only find magnetospheric electrons and these are accelerated to form a beam over the acceleration region which extends to altitudes $\sim$1 R_E above ℓ_c. The upward $E_\parallel$ in this region has a value of $\sim$mV/m, and peaks at ℓ_E which is at an altitude of a couple of density scale heights above ℓ_c.

[18] The region $\ell_c < \ell < \ell_m$ corresponds to the F region. Trapped ionospheric electrons are found here as well as a keV beam of magnetospheric electrons. There is no significant energization of the beam in this region. The boundary between the E and F region occurs at ℓ_m, and the field-aligned current leaving the F region will be closed by perpendicular currents in the E region, although we do not describe that part of the circuit in the present calculation.

3.2. Boundary Conditions and Constraints

[19] The boundary condition on $F(\ell, v_\parallel)$ is imposed at ℓ_m, where both ionospheric electrons and energetic magnetospheric electrons are present. The ionospheric population is trapped and characterized, for simplicity, by the top-hat distribution $F(\ell_m, v_\parallel) = F_1$, $-a_m \leq v_\parallel \leq + a_m$. This distribution could be replaced by a Gaussian, but has the great advantage of keeping the following results analytical and permitting considerable insight into the acceleration

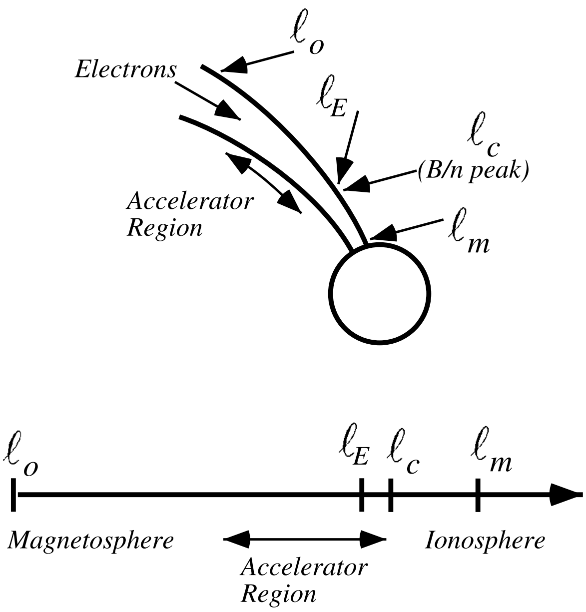

Figure 2. Schematic of the model equilibrium converging field. For an upward current the electrons travel downwards and must carry an increasing current density ($\propto B$) as they approach the Earth. Some important locations are: ℓ_0, a reference point far out in the magnetosphere; ℓ_E, the point where $E_\parallel$ has its greatest magnitude; ℓ_c, the peak of B/n; and ℓ_m, the base of the ionospheric F region. The region in which most of the electron acceleration takes place extends for about 1 R_E beyond ℓ_c.

process. In a similar spirit the magnetospheric beam has a speed $b_m \gg a_m$, and is represented by the distribution $F(\ell_m, v_\parallel) = F_2$, $b_m - \varepsilon_m \leq v_\parallel \leq b_m + \varepsilon_m$, the beam width being $2\varepsilon_m$. $F(\ell_m, v_\parallel)$ is zero elsewhere.

[20] The electrons are much more mobile than the ions, so we adopt *Rönnmark*'s [1999] suggestion of taking the ion density to be constant and given by equation (3). Formally this assumption gives us the leading (short timescale) solution to our problem. Since the plasma is quasi-neutral on the timescales of interest, we shall require the electron number density given by equation (9a) to be equal to that of the ions in equation (3). Indeed, a general $\Phi(\ell)$ will map F into the domain $\ell_0 < \ell < \ell_m$ in such a way that quasi-neutrality is not met. Thus relating equations (3) and (9a) in this way imposes a fundamental constraint on our solution.

[21] Another important constraint is that of current continuity. Over the acceleration region current is predominantly field-aligned, and its strength is focused by the convergence of field lines: $\nabla \cdot \mathbf{j} = 0$ in curvilinear coordinates may be expressed as

$$\frac{j(\ell)}{B(\ell)} = \frac{j_m}{B_m} \tag{13}$$

where $j_m = j(\ell_m)$ (j being the field-aligned component of the current density) and $B_m = B(\ell_m)$, and is used to constrain the integral in equation (9b).

3.3. Electrostatic Potential Solution

[22] The above boundary conditions and constants can be used to determine $\Phi(\ell)$. For a general $\Phi(\ell)$ we have

$$\frac{1}{2}v_\parallel^2 = \Phi(\ell) + W_0/m \tag{14}$$

where W_0 is the total energy of the electron. An electron with speed $v_{\parallel m}$ at ℓ_m will have a different speed at ℓ given by

$$v_\parallel = \pm\sqrt{v_{\parallel m}^2 + 2\Delta\Phi(\ell)} \tag{15}$$

where $\Delta\Phi$ is the change in normalized potential,

$$\Delta\Phi(\ell) = \Phi(\ell) - \Phi(\ell_m) \tag{16}$$

Ionospheric electrons are confined to regions where $-a_m^2/2 \leq \Delta\Phi(\ell) \leq 0$. Defining ℓ_c by $\Delta\Phi(\ell_c) = -a_m^2/2$, this condition identifies the region $\ell_c \leq \ell \leq \ell_m$, within which the boundary speed a_m maps to $a(\ell) = \pm\sqrt{a_m^2 + 2\Delta\Phi(\ell)}$. Thus the ionospheric electrons make a contribution to equation (9a) of $2F_1\sqrt{a_m^2 + 2\Delta\Phi(\ell)}$.

[23] Similarly, the beam speed (b_m) and width (ε_m) map, using equation (15), to

$$b(\ell) = \sqrt{b_m^2 + 2\Delta\Phi(\ell)} \tag{17a}$$

$$\varepsilon(\ell) \approx \frac{b_m}{b(\ell)}\varepsilon_m \equiv \frac{b_m\varepsilon_m}{\sqrt{b_m^2 + 2\Delta\Phi}} \tag{17b}$$

contributes $\approx 2F_2b_m\varepsilon_m/\sqrt{b_m^2 + 2\Delta\Phi(\ell)}$ to the integral in equation (9a). Note we have neglected thermal effects in the beam ($\varepsilon_m \ll b_m$), and these electrons access the larger domain $-b_m^2/2 \leq \Delta\Phi \leq 0$ compared to their ionospheric counterparts. Thus equation (9a) requires in the ionosphere ($-a_m^2/2 \leq \Delta\Phi \leq 0$)

$$\frac{n(\ell)}{B(\ell)} = 2F_1\sqrt{a_m^2 + 2\Delta\Phi} + \frac{2F_2b_m\varepsilon_m}{\sqrt{b_m^2 + 2\Delta\Phi}} \tag{18a}$$

and in the magnetosphere ($-b_m^2/2 \leq \Delta\Phi < -a_m^2/2$)

$$\frac{n(\ell)}{B(\ell)} = \frac{2F_2b_m\varepsilon_m}{\sqrt{b_m^2 + 2\Delta\Phi}}. \tag{18b}$$

Noting that the trapped ionospheric electrons carry zero net current, the constraint (equation (9b)) becomes

$$\frac{j(\ell)}{B(\ell)} \approx -2eF_2b(\ell)\varepsilon(\ell) \equiv -2eF_2b_m\varepsilon_m \tag{19}$$

Note the final relation, found by employing equation (17b), is exact even if ε_m/b_m is not small and confirms that $j(\ell)/B(\ell)$ is indeed constant. This is consistent with the relation in equation (13), which together with equation (19) gives

$$2F_2b_m\varepsilon_m = -j_m/(eB_m) \tag{20}$$

(Assuming that $b_m > 0$ we require j_m to be negative, i.e., j flows antiparallel to **B**.)

[24] A further relation, for F_1, is found by evaluating equation (18a) at ℓ_m (i.e., $\Delta\Phi = 0$) in conjunction with equation (20),

$$F_1 = \left(\frac{n_m}{B_m} + \frac{j_m}{eB_m b_m}\right)\frac{1}{2a_m} \tag{21}$$

where $n_m = n(\ell_m)$. Substituting the two relations above into equations (18a) and (18b) yields, in the ionosphere ($-a_m^2/2 \le \Delta\Phi \le 0$),

$$\frac{n(\ell)}{B(\ell)} = \left(\frac{n_m}{B_m} + \frac{j_m}{eB_m b_m}\right)\sqrt{1 + 2\Delta\Phi/a_m^2} - \frac{j_m}{eB_m\sqrt{b_m^2 + 2\Delta\Phi}} \tag{22a}$$

and in the magnetosphere ($-b_m^2/2 \le \Delta\Phi < -a_m^2/2$)

$$\frac{n(\ell)}{B(\ell)} = \frac{-j_m}{eB_m\sqrt{b_m^2 + 2\Delta\Phi}}. \tag{22b}$$

Evidently $\Delta\Phi = -a_m^2/2$ corresponds to a critical value of the potential (say, $\Delta\Phi_c$) for which $n = n_c$ and $B = B_c$;

$$\frac{n_c}{B_c} = \frac{-j_m}{eB_m\sqrt{b_m^2 - a_m^2}}. \tag{23}$$

Performing a series expansion of equation (22a) in the variable $\delta\Phi = \Delta\Phi - \Delta\Phi_c \ge 0$ we find the change in (n/B) is

$$\delta\left(\frac{n}{B}\right) \approx \left(\frac{n_m}{B_m} + \frac{j_m}{eB_m b_m}\right)\frac{\sqrt{2}}{a_m}\sqrt{\delta\Phi} + \frac{j_m}{eB_m\left(b_m^2 - a_m^2\right)^{3/2}}\delta\Phi \tag{24}$$

so as $\Delta\Phi$ increases slightly from $\Delta\Phi_c$, n/B will increase. Similarly, inspection of equation (22b) shows that as $\Delta\Phi$ decreases slightly below $\Delta\Phi_c$, n/B will also increase. Thus $\Delta\Phi_c = \Delta\Phi(\ell_c)$ occurs at the minimum of n/B – i.e., ℓ_c is located at the peak of B/n. This result means that ionospheric electrons are trapped earthward of the B/n peak.

[25] Note that we have not assumed *a priori* that the ionospheric electrons are confined to altitudes below the B/n peak. This is a general property of our model, although somewhat surprising at first sight. For example, suppose we increase the energy of the ionospheric electron distribution to a'_m ($a'_m > a_m$). It would not be unreasonable to think that the ionospheric electrons would now escape to altitudes beyond the B/n peak. However, this is not the case: In terms of $\Delta\Phi$, the electrons are confined to the region $\Delta\Phi'_c \le \Delta\Phi < 0$, where $\Delta\Phi'_c = -a_m'^2/2$. The new version of equation (22b) shows that for $\Delta\Phi$ slightly less than $\Delta\Phi'_c$, n/B must increase. Moreover, the new version of equation (24) shows that if $\Delta\Phi$ is slightly larger than $\Delta\Phi'_c$, n/B must again increase. Thus we conclude that $\Delta\Phi'_c$ is coincident with the peak of B/n, and the ionospheric electrons are always trapped below this peak. We note that this property could be modified if a different ionospheric electron distribution was chosen, e.g., a Maxwellian. However, it is still likely that the vast majority of ionospheric electrons would be confined to being below the B/n peak.

[26] Our aim is to solve for $\Delta\Phi$ (ℓ), and we proceed by inverting equations (22a) and (22b) to get $\Delta\Phi$ in terms of n/B, which is a known function of ℓ. This is straightforward in the magnetosphere, where $-b_m^2/2 \le \Delta\Phi < -a_m^2/2$. For the ionospheric interval $-a_m^2/2 \le \Delta\Phi \le 0$ we use $n/B = n_c/B_c + \delta(n/B)$ (see equation (24)) with $\delta\Phi = \Delta\Phi - \Delta\Phi_c$) and note that the final term in equation (24) may be neglected: This term is smaller than the leading term at $\Delta\Phi = 0$ ($\ell = \ell_m$, $\delta\Phi = a_m^2/2$) by a factor of the order of $(a_m/b_m)^2(n_c/B_c)/(n_m/B_m) \sim 10^{-6}$ if $(a_m/b_m)^2 \sim 10^{-3}$, so this is an excellent approximation. (For the parameters given in section 2.1, $n_c/n_m = 1.14 \times 10^{-3}$ and $B_c/B_m = 0.26$.) Incorporating equation (23), the resulting $\Delta\Phi$ in the ionosphere ($\ell_c \le \ell \le \ell_m$) is given by

$$\Delta\Phi(\ell) = \frac{a_m^2}{2}\left(\left[\frac{\frac{n(\ell)}{B(\ell)} - \frac{n_c}{B_c}}{\frac{n_m}{B_m} - \frac{n_c}{B_c}\sqrt{1 - a_m^2/b_m^2}}\right]^2 - 1\right) \tag{25a}$$

while that in the magnetosphere ($\ell_0 \le \ell < \ell_c$) is

$$\Delta\Phi(\ell) = \frac{b_m^2}{2}\left(\left(\frac{n_c}{B_c}\cdot\frac{B(\ell)}{n(\ell)}\right)^2\left(1 - \frac{a_m^2}{b_m^2}\right) - 1\right) \tag{25b}$$

A plot of $\Delta\Phi$ is given in Figure 3 in terms of $s = \ell_m - \ell$, i.e., the distance along the field line from the base of the F region. Figure 3b shows the potential drop (due to the ambipolar electric field) across the ionosphere is very small compared to that over the magnetosphere as expected by *Rönnmark* [1999] and *Wright et al.* [2002].

3.4. Parallel Electric Field

[27] According to equation (10), $E_\parallel = -d\phi/d\ell = -(m/e)\, d\Delta\Phi/d\ell$ where all derivatives are understood to be taken along the field line. Differentiating equation (25b) shows that in the magnetosphere $E_\parallel \propto (B/n)d(B/n)/d\ell$, while in the ionosphere (from equation (25a)) $E_\parallel \propto (n/B - n_c/B_c)\, d(n/B)/d\ell$, and so $E_\parallel = 0$ at the peak of B/n, i.e., ℓ_c. This is the point where the magnetospheric electrons have finished being accelerated, but it is obviously not the location of the largest $E_\parallel$. Inspection of Figure 3 shows that the maximum of $E_\parallel$ occurs in the magnetosphere and is located at ℓ_E, where $d^2(B^2/n^2)/d\ell^2 = 0$. Differentiating equations (25a) and (25b) yields $E_\parallel$ explicitly. In the magnetosphere ($\ell_0 \le \ell < \ell_c$)

$$\frac{E_\parallel}{mb_m^2/e} = -\left(1 - \frac{a_m^2}{b_m^2}\right)\left(\frac{n_c}{B_c}\right)^2\left(\frac{B}{n}\right)\cdot\frac{d}{d\ell}\left(\frac{B}{n}\right) \tag{26a}$$

while in the ionosphere ($\ell_c \le \ell \le \ell_m$), after neglecting n_c/B_c compared to n_m/B_m,

$$\frac{E_\parallel}{mb_m^2/e} = \frac{a_m^2}{b_m^2}\left(\frac{n}{B} - \frac{n_c}{B_c}\right)\left(\frac{B_m}{n_m}\cdot\frac{n}{B}\right)^2\frac{d}{d\ell}\left(\frac{B}{n}\right) \tag{26b}$$

Figure 4 shows the variation of $E_\parallel(s)$ above the Earth ($s = \ell_m - \ell$).

4. Interpretation

[28] The solution given in the previous section yields considerable insight into the physical processes leading to

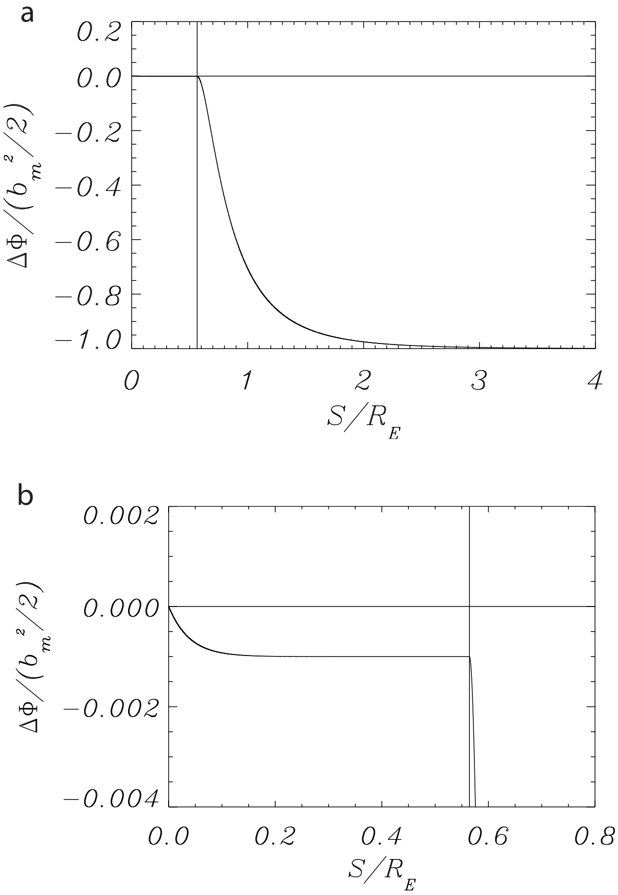

Figure 3. The variation of $\Delta\Phi$ with $s = \ell_m - \ell$ over the range (a) $0 < s/R_E < 4$, and (b) $0 < s/R_E < 0.8$. The vertical line ($s/R_E = 0.56$) marks the location of the B/n peak, below which ionospheric electrons are trapped and a small ambipolar potential exists.

the formation of the energetic electron beam. We begin exploring this solution via the B/n curve which is central to the whole problem.

4.1. Densities and Scale Lengths

[29] Figure 1 shows the peak of B/n occurs (at $s_c/R_E = 0.56$) where $d(B/n)/d\ell = 0$, which may be written

$$\frac{B'}{B} = \frac{n'}{n} \tag{27}$$

($'$ denotes $d/d\ell = -d/ds$.) Evidently the scale lengths of B and n play an important role in determining the location of the peak. We note that in general the vertical density scale height (h) used in equation (3) is slightly different from the field-aligned scale height ($h_\|$) due to the inclination of the field line. For a dipolar field

$$\frac{h_\|}{h} = -\frac{d\ell}{dr} = \frac{\sqrt{1+3\sin^2\theta}}{2\sin\theta} \tag{28}$$

At altitudes corresponding to the peaks of B/n and $E_\|$ we find $h_\|/h \approx 1.025$, since the field is close to radial. Also, at the B/n peak the scale length of the magnetic field is $L_B = B/B' = 0.52\ R_E$, so taking $h = 400$ km gives $h/R_E = 0.0625$ - i.e., the density varies on a scale that is smaller than that of the magnetic field variation by an order of magnitude.

[30] Given the density variation in equation (3), and approximating the field as vertical near the B/n peak, we can express the density near ℓ_c as

$$n(\ell) = n_0 + \delta n_c \exp[(\ell - \ell_c)/h] \tag{29}$$

where $\delta n_c = n(\ell_c) - n_0$ is the excess ion density above the magnetospheric value (n_0) at ℓ_c. Thus $n'(\ell_c) = \delta n_c/h$, and equation (27) implies

$$\frac{\delta n_c}{n_0} = \frac{1}{B_c/(B_c'h) - 1} \approx h\frac{B_c'}{B_c} \tag{30}$$

indicating that $\delta n_c/n_0 \approx 0.14$. This means that the downgoing electron beam is accelerated only until it encounters the very top of the ionosphere and experiences a small enhancement of n by 14%. This is very small compared to the final density enhancement at ℓ_m of several orders of magnitude.

[31] As mentioned previously, the point where $E_\| = 0$ coincides with the peak of B/n, and occurs at ℓ_c. $E_\|$ has its peak at ℓ_E where $d^2(B^2/n^2)/d\ell^2 = 0$, and we can determine the excess density ($\delta n_E = n(\ell_E) - n_0$) there by noting that when $h/L_B \ll 1$ and $\delta n_E/n_0 \ll 1$ the leading terms of this second derivative require, at ℓ_E,

$$\frac{B^{2''}}{n^2} \approx 2B^2\frac{n''}{n^3} \tag{31}$$

Using a similar expansion to equation (29) about ℓ_E, equation (31) gives

$$\frac{\delta n_E}{n_0} \approx \frac{h^2}{2}\cdot\frac{B_E^{2''}}{B_E^2} \tag{32}$$

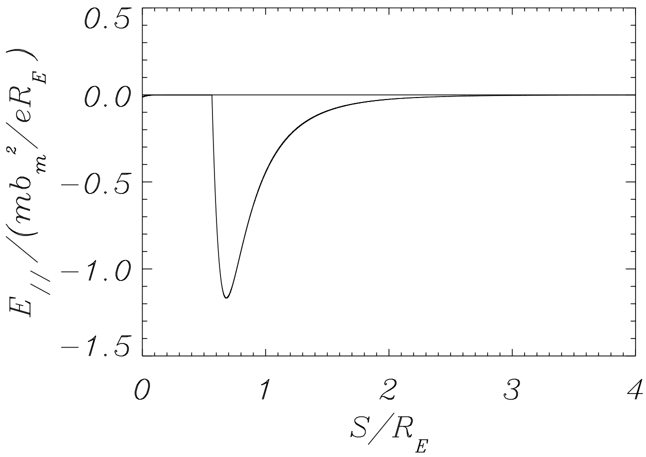

Figure 4. The variation of $E_\|$ with s: $E_\|$ falls to zero at the B/n peak ($s/R_E = 0.56$), and has its peak in the magnetosphere where $d^2(B^2/n^2)/ds^2 = 0$ (i.e., at ($s/R_E = 0.68$). The parallel electric field in the magnetosphere ($s/R_E > 0.56$) is associated with the acceleration of electrons to carry the field-aligned current, while in the ionosphere ($s/R_E < 0.56$) it is predominantly an ambipolar field ensuring the electrons maintain quasi-neutrality.

since $B^2/B^{2''} \approx 0.07\ R_E^2$ at the altitude we are interested in, we find $\delta n_E/n_0 \approx 0.03$, which is in good agreement with the exact value of 0.023. Thus the peak value of $E_\parallel$ occurs when the ionospheric ions make up only a few percent of the total number density. This will occur at slightly higher altitudes than ℓ_c. Indeed, if $\Delta\ell = \ell_c - \ell_E$, evaluating equation (29) at ℓ_E gives

$$\Delta\ell \approx h \ln\left(\frac{\delta n_c}{\delta n_E}\right) \approx h \ln\left(\frac{2BB'}{hB^{2''}}\right) \quad (33)$$

to leading order in h/L_B. For the parameters in our calculation, equation (33) gives $\Delta\ell/h \approx 2$, so $E_\parallel$ has its peak a couple of density scale heights above the B/n peak. This is in good agreement with the exact value of 2.3 (s_E = 0.68).

4.2. Electron Beam Speed

[32] Conservation of total energy (equation (12)) means that as $\Delta\Phi$ increases (Figure 3) so will the speed of the electron beam. It is convenient to define the current density as

$$j(\ell) = -n^\star(\ell)eb(\ell) \quad (34)$$

where $n^\star(\ell)$ is the number density of electrons that contribute to the current. In the magnetosphere ($\ell_0 \le \ell < \ell_c$) $n^\star(\ell) = n(\ell)$ since only magnetospheric electrons moving with the beam speed (b) exist in this region. However, in the ionosphere ($\ell_c \le \ell \le \ell_m$) $n^\star(\ell)$ represents the number density of the current carrying beam electrons which will generally be much less than the density of ionospheric electrons.

[33] The interpretation is that at very high altitudes (say, $s > 2R_E$) the beam moves slowly to carry the weak current density ($b = -j/(ne)$). Above the B/n peak, ($s_c < s < 2R_E$) where $n^\star \approx n_0$, the increase of j on approaching the Earth is met by an increase in beam speed, b - particularly where $\Delta\Phi$ changes significantly (i.e., $E_\parallel$ is large). This is precisely the feature predicted by *Rönnmark* [1999] and supported by *Wright et al.* [2002].

[34] The situation is quite different earthward of the B/n peak where only a fraction $n^\star/n$ of the electrons carry the current. The maximum beam speed (b_m) and current density (j_m) are not independent as found in equation (23) which we use to write the first equality below

$$\frac{-j_m}{eB_m b_m} = \frac{n_c}{B_c}\sqrt{1 - a_m^2/b_m^2} = \frac{n_m^\star}{B_m} \quad (35)$$

Note that n_c/B_c is a property of the equilibrium alone, and does not depend upon the current strength, so equation (35) gives a relation between current density and beam speed, or equivalently energy. Given that $a_m^2/b_m^2 \ll 1$, and evaluating equation (13) at ℓ_c, equation (35) gives the energy of the beam as

$$\frac{1}{2}mb_m^2 \approx \frac{mj_c^2}{2n_c^2e^2} \quad (36)$$

where $j_c = j(\ell_c)$.

[35] The second equality in equation (35) follows from evaluating equation (34) at ℓ_m and substituting for j_m in the first expression in equation (35). At ℓ_c we only have beam electrons, so $n_c \equiv n_c^\star$, and recalling that $a_m^2/b_m^2 \ll 1$ (equation (35)) implies

$$\frac{n_c^\star}{B_c} \approx \frac{n_m^\star}{B_m} \quad (37)$$

and reveals how the increase in current density between ℓ_c and ℓ_m is achieved: The number density of beam electrons increases proportional to B and so will naturally carry the current (which also increases $\propto B$) without requiring further acceleration. Thus the beam speed does not change significantly over the ionosphere and the potential remains approximately constant here (see Figure 3).

4.3. Current-Voltage Relation and $E_\parallel$

[36] *Rönnmark* [1999] found the voltage, or potential, along a field line varied as j^2, rather than j as in *Knight*'s [1973] calculation. Subsequently, *Rönnmark* [2002] showed how the current-voltage relation depended crucially upon the boundary conditions imposed on the distribution function. Our boundary conditions produce the $\Delta\Phi \propto j^2$ relation: Evaluating equation (25b) at ℓ_0 (where $B_0/n_0 \ll B_c/n_c$) and employing equation (36) yields the total potential drop along the field line (recall $\phi = (m/e)\Phi$)

$$\phi_m - \phi_o \approx \frac{mj_c^2}{2e^3n_c^2} \quad (38)$$

which is in agreement with *Rönnmark* [1999]. Note that the plausible physical justifications that he gave in some steps of his calculation are supported by our more detailed electron distribution function analysis.

[37] The small ambipolar $E_\parallel$ that traps the ionospheric electrons has its maximum value at $s = 0$ ($\ell = \ell_m$) where (using equation (26b)) and noting $B_m/n_m \ll B_c/n_c$)

$$E_\parallel(s = 0) \approx -\frac{m}{e}\cdot\frac{a_m^2}{h} \quad (39)$$

For the parameters we have chosen $(n_c/B_c)/(n_m/B_m) = 0.004$ and $E_\parallel(s = 0) \times (eR_E/mb_m^2) \approx -0.016$, in good agreement with the exact value of -0.013 in Figure 4.

[38] In the magnetosphere, equation (26a) may be used with the first equality in equation (35) and equation (13) to eliminate b_m and provide an $E_\parallel$–j relation, i.e., an Ohm's law in the magnetosphere of

$$E_\parallel(\ell_0 < \ell < \ell_c) = -\frac{m}{e^3}\cdot\frac{j}{n}\nabla_\parallel\left(\frac{j}{n}\right) \quad (40)$$

which is identical to (35) of *Wright et al.* [2002] suggesting that the electron fluid model is a reasonable approximation in the magnetosphere and supports their claim, and that of *Rönnmark* [1999], that the convective derivative dominates the electron dynamics. Integrating equation (40) along the magnetospheric section of field line gives, to leading order, the potential drop found in equation (38).

[39] The magnitude of $E_\parallel$ in the magnetosphere can be estimated by evaluating equation (26a) at ℓ_E ($s_E = 0.68\ R_E$)

$$E_\parallel(\ell_E) \approx -\frac{mb_m^2}{e}\left(\frac{n_c}{B_c}\cdot\frac{B_E}{n_E}\right)^2\frac{B_E'}{B_E} \tag{41}$$

where we have taken $a_m^2/b_m^2 \ll 1$ and neglected the derivative of n compared to that of B at ℓ_E since $\delta n_E/n_E \ll hB'/B$. Noting that $B'/B = 0.7\ R_E$ at ℓ_E and $(n_cB_E/n_EB_c)^2 = 0.80$, equation (41) suggests the peak of $E_\parallel$ in Figure 4 is -1.1, which agrees well with the exact value of -1.2.

[40] The properties of the equilibrium we assumed in our calculation may be summarized as follows. At ℓ_0, $\ell = 0.0\ R_E$, $s_0 = 12.79\ R_E$, $B = B_0$, $n = n_0$; At ℓ_E, $\ell = 12.11\ R_E$, $s_E = 0.681\ R_E$, $B_E/B_0 = 402.64$, $n_E/n_0 = 1.023$, $(n_E/B_E)/(n_0/B_0) = 0.0254$, $L_B = 0.7\ R_E$; At ℓ_c, $\ell = 12.23$, $s_c = 0.56\ R_E$, $B_c/B_0 = 500.6$, $n_c/n_0 = 1.14$, $(n_c/B_c)/(n_0/B_0) = 0.00227$ $L_B = 0.52\ R_E$; At ℓ_m, $\ell = 12.79R_E$, $s_m = 0.0\ R_E$, $B_m/B_0 = 1923.5$, $n_m/n_0 = 1000.0$, $(n_m/B_m)/(n_0/B_0) = 0.5199$. Also, $(B_E/B_c)^2 = 0.0647$, $(n_cB_E/n_EB_c)^2 = 0.805$, and $B_E^2/B_E^{2\prime\prime} = 0.071\ R_E^2$.

4.4. Origin of $E_\parallel$

[41] Early quasi-neutrality calculations such as *Chiu and Schultz* [1978] and *Stern* [1981] focused upon the equilibrium of a hot diffuse magnetospheric plasma and a cold dense ionospheric plasma. Their calculations included both electron inertia and magnetic mirroring. *Stern* [1981] found double layers to be a common feature of his solutions, whereas *Chiu and Schultz* [1978] found $E_\parallel$ had a profile and location quite similar to ours. Although Chiu and Schultz did not impose any condition on the field-aligned current, this quantity would in general be non-zero and it would be interesting to see if the form of their $\Delta\Phi - j$ relation also agreed with ours. The corresponding relation for *Stern*'s double layers was linear, and he explains this in terms of the dominance of the mirroring effect [see also *Knight*, 1973].

[42] On the basis of our $\Delta\Phi \propto j^2$ relation being the same as that of *Rönnmark* [1999], we anticipate that electron inertia will dominate the mirroring effect in our calculation. Indeed, we certainly have electron inertia in the equation of motion for an electron,

$$m\frac{d\mathbf{u}}{dt} = -e(\mathbf{E} + \mathbf{u}_\wedge\mathbf{B}) \tag{42}$$

Here $\mathbf{u}$ is the velocity of an electron, rather than its guiding center drift velocity, $\mathbf{v}$, as in equation (4).

[43] The concept of mirroring is best illustrated by neglecting gyrophase information and considering the motion of the guiding center, having velocity $\mathbf{v}$. The parallel component of the guiding center equation of motion [e.g., *Clemmow and Dougherty*, 1969] may be rewritten as

$$E_\parallel = -\frac{\mu}{e}\frac{dB}{d\ell} - \frac{m}{e}\frac{dv_\parallel}{dt} \tag{43}$$

For example, in the absence of $E_\parallel$ the terms on the r.h.s. must cancel: the guiding center is repelled from regions of increasing field strength and may be mirrored, or reflected. The dominance of the mirroring term over the inertial term produces a linear $\Delta\Phi \propto j$ relation [*Knight*, 1973].

[44] In the present calculation the distributions have $\mu \approx 0$, so the electron inertial term dominates the mirror term. As shown in the previous subsection, this produces a $\Delta\Phi \propto j^2$ relation. We can move to a cruder description of the electrons by considering the average guiding center drift which corresponds to the center of mass velocity of the electron fluid (**V**), and is the approximation employed by both *Rönnmark* [1999] and *Wright et al.* [2002]. These studies also stressed the importance of electron inertia and find identical $E_\parallel - j$ relations to ours (equations (38) and (40)) in the magnetosphere, confirming that our $E_\parallel$ is indeed associated with electron inertia.

[45] It is surprising that the present steady $E_\parallel - j$ relation (equation (40)) agrees exactly with the time-dependent Alfvén wave result of *Wright et al.* [2002]. Recall that from the particle viewpoint 1 keV electrons will traverse the acceleration region on a timescale of the order of 1 s. Thus, we argued that on the Alfvén wave period (a few 100 s) the electrons would experience quasi-static Alfvén wave fields, and so neglected time variations. From the electron fluid perspective *Wright et al.* [2002] showed that

$$E_\parallel \approx -\frac{m}{e}\left(\frac{\partial V_\parallel}{\partial t} + \left(V_\parallel\nabla_\parallel\right)V_\parallel\right) \tag{44}$$

In the acceleration region they found the high electron speed of 1 keV electrons and low Alfvén wave frequency resulted in $(V_\parallel\nabla_\parallel)V_\parallel$ exceeding $\partial V_\parallel/\partial t$ by two or three orders of magnitude. This confirms the appropriateness of adopting a quasi-steady solution in which $\partial/\partial t$ is neglected. Indeed, omitting $\partial V_\parallel/\partial t$ from equation (45) and noting that $j = -neV_\parallel$ we recover our present result (equation (40)). Of course, there are also cases where waves have a steady driver or may be viewed in a frame for which $\partial/\partial t$ is identically zero [e.g., *Rönnmark*, 1999]. In such a situation the omission of $\partial/\partial t$ is not an approximation.

5. Summary

[46] We have presented a distribution function solution for electrons that are accelerated into the ionosphere to carry the field-aligned current necessary for coupling the magnetosphere and ionosphere. Our formulation has an advantage over electron fluid models in that we can identify the accelerated magnetospheric beam population and the cold ionospheric population that it penetrates.

[47] The cold ionospheric electrons ($kT \sim 1$ eV) are trapped below the B/n peak by a small ambipolar electric field (equation (39)) of magnitude $\sim\mu$V/m. The peak of B/n (i.e., $d(B/n)/d\ell = 0$) has $E_\parallel = 0$, and at higher altitudes a much larger $E_\parallel$ accelerates the magnetospheric electron beam over a distance of $\sim 1\ R_E$. The maximum $E_\parallel$ in this region occurs not at the B/n peak, but where $d^2(B^2/n^2)/d\ell^2 = 0$, which is a couple of density scale heights beyond the B/n peak. The density over this acceleration region is essentially just due to magnetospheric ions ($\delta n_E/n_0 \approx 0.02$, $\delta n_c/n_0 \approx 0.14$). For a 1 keV beam the maximum magnitude of $E_\parallel$ (equation (41)) is 0.5 mV/m, and for a 10 keV beam we find $E_\parallel \approx 5$ mV/m. These values agree well with anticipated values.

[48] Once the beam enters the ionosphere there is no significant additional energization of the beam, and the

increase in current density within the ionosphere is achieved by the natural focusing of the beam by the magnetic field. Indeed, these results support the modification *Wright et al.* [2002] suggested on qualitative grounds to the $E_{\parallel}$ they found from an electron fluid treatment. (Their cartoon of $E_{\parallel}$ and parallel beam speed in their Figure 5 agrees exactly with our findings.) However, our solution has revealed a detail that they did not anticipate: They argued that the natural focusing of the beam would only occur once $n^{\star}/n \ll 1$ (i.e., the number density of the beam is much less than n) since then a small adjustment to the abundant ionospheric population could accommodate the additional beam number density and preserve quasi-neutrality. However, our solution shows that the number density of the beam in the outer magnetosphere is n_0, increases to $1.02n_0$ at ℓ_E, and then to $1.14n_0$ at ℓ_c. Thereafter ionospheric electrons are encountered, yet the geometrical focusing (equation (37)) operates at once: It is not necessary to wait for $n^{\star}/n \ll 1$.

[49] Viewing the beam as a current element we find the voltage drop along the field line varies as j^2, as predicted by *Rönnmark* [1999]. The recent study by *Rönnmark* [2002] suggests that this form of Ohm's law is a result of our boundary conditions. An important omission from our calculation is a full pitch angle distribution: *Rönnmark* [2002] finds that the associated thermal and mirroring effects modify our Ohm's law by increasing ϕ and $E_{\parallel}$ by a factor of 4. Thus our study provides a useful benchmark with which to compare calculations including full thermal effects, and also those employing the electron fluid approximation. The behavior of electrons for downward current (i.e., upgoing electrons) will be very different from the solution here, and will be addressed in a separate paper.

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