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The absolute and convective instability of the magnetospheric flanks

Andrew N. Wright, Katharine J. Mills, and Michael S. Ruderman
Department of Mathematical and Computational Sciences, University of St. Andrews, St. Andrews, Fife, Scotland

Leonid Brevdo
Ecole Supérieure de Mechanique de Marseille, Université de la Mediterranée, Marseille, France

Abstract. Despite the existence of flank waveguide modes which are Kelvin-Helmholtz unstable, the flanks of the terrestrial magnetosphere are observed to be remarkably stable and free of nonlinear disturbances. We suggest the explanation may be found in a more detailed stability analysis which shows that localized disturbances are convectively unstable in the Earth's rest frame. This means that as a wave packet grows and broadens, it also propagates at a sufficiently high speed so it is convected away leaving ultimately no disturbance at any fixed point in space (as $t \to \infty$). We estimate that the magnetopause surface wave has an e-folding length of the order of an Earth radius and soon becomes nonlinear, resulting in a magnetopause boundary layer [e.g., *Manuel and Samson*, 1993]. In contrast, the waveguide modes (which penetrate deep into the body of the magnetosphere) should grow by no more than a factor of about e as they propagate around the flanks to the tail. This also explains why theorists have had such success at modeling basic ULF waveguide processes with linear theory and why nonlinear waves in, or disruptions to, the body of the magnetospheric flanks are not observed: Wavepackets may grow by only a small amount as they propagate into the tail. Ultimately, they leave the flank undisturbed and with the appearance of stability, although they are actually convectively unstable.

1. Introduction

The magnetopause Kelvin-Helmholtz (KH) instability was first suggested by *Dungey* [1954]. Subsequent treatments in a compressible plasma followed *Fejer* [1964] and *Dungey* [1967] and the marginal stability calculation of *Southwood* [1968]. Further studies of normal modes by *McKenzie* [1970], *Yumoto and Saito* [1980], *Walker* [1981], and *Pu and Kivelson* [1983] confirmed that the magnetopause, particularly on the flanks, would be generally unstable. Field line tension may provide some stabilization, as may ionospheric "line tying" [*Miura*, 1992a]. Nevertheless, the surface modes considered in the above studies grow rapidly and become nonlinear, and this is the ultimate fate of linear surface mode perturbations. The nonlinear phase has been investigated numerically by *Muira* [1987, 1992b] in a periodic domain and was shown to saturate. Relaxing the requirement of periodicity leads to vortex merging [*Muira*, 1995] or the continual growth of a vortex to the dimensions of the simulation domain [*Wu*, 1986]. *Manuel and Samson* [1993] showed how the nonlinear phase grew more slowly than the linear phase, and they suggest that the broadening of the low-latitude-bounday-layer could be accounted for by the nonlinear disruption associated with the Kelvin-Helmholtz surface wave.

The situation of the surface wave on the magnetopause is clear: It is unstable and grows until it is nonlinear. While doing so, it disrupts the vortex sheet that supported the linear wave, destroys the original equilibrium, and produces a broad turbulent shear layer [*Manuel and Samson*, 1993]. There are many observations of magnetopause surface waves [see *Belmont and Chanteur*, 1989, and *Miura*, 1999, and references therein]. *Miura* [1999] has shown that these observations (occurring at distances of 5 R_E or greater from the subsolar point) are consistent with a nonlinear development and vortex pairing.

Kelvin-Helmholtz modes are commonly thought to be surface waves, and indeed, this is the case for incompressible hydrodynamics where such instabilities were first considered. Surface waves are characterized by a rapid spatial decay away from both sides of the shear flow layer. Although the classical surface wave is strictly a solution about a discontinuity in some equilibrium quantity (e.g., the flow), similar solutions exist when

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the equilibrium changes over a narrow layer, and we shall also term these surface modes. Compressible fluids and MHD with noninfinite domains (i.e., waveguides or channels) permit modes which have a spatial structure that is oscillatory normal to the vortex layer. These are sometimes called "body" or "waveguide" (in contrast to "surface") modes. Recently, it has been shown that such modes can also be driven unstable by the free energy of the shear layer [*Fujita et al.,* 1996; Mann et al., 1999; Mills et al., 1999]. It is quite legitimate to refer to these as Kelvin-Helmholtz-driven modes, although this is a more general usage than that in the incompressible fluid literature. To avoid any confusion, we shall use the terms KH surface and KH waveguide/body modes when appropriate.

Calculations of the normal modes of the waveguide formed by the magnetospheric flanks show that these may be unstable when $V_0 > c_s + V_A$, where V_0 is the sheath flow speed, c_s is the sheath sound speed, and V_A is the magnetospheric Alfvén speed just inside the magnetopause [*Mann et al.*, 1999; *Mills et al.*, 1999]. This relation suggests that waveguide modes are unstable when V_0 exceeds roughly 500 km s^{-1} and is in excellent agreement with that of *Engebretson et al.* [1998], who reported enhanced Pc5 activity for solar wind speeds in excess of 500 km s^{-1}.

Given that waveguide modes are sometimes unstable, it would seem reasonable to expect them to grow exponentially with time until they become nonlinear. In doing so, the waves would disrupt the body of the magnetosphere, suggesting that it could be destroyed on occasions or disturbed by nonlinear waves. This has never been observed, and satellites traversing the middle and outer magnetosphere see a robust equilibrium about which there may be small fluctuations. It would appear that waveguide modes generally maintain a small amplitude, making them somewhat elusive in observations. Although *Kivelson et al.* [1984] observed a coherent compressional wave across a range of L shells, such examples are extremely rare. The majority of evidence for waveguide/cavity modes is indirect, and normally focuses on the field line resonances (FLRs) to which they couple [*Kivelson and Southwood*, 1985; *Lee and Lysak*, 1989; *Wright*, 1994: *Rickard and Wright*, 1994]. For example, *Yeoman et al.* [1997] observed the detailed growth of an FLR and found it to be in excellent agreement with a fast mode driver [*Mann et al.*, 1995]. In addition, *Crowley et al.* [1987] observed the damping rate of an FLR to be significantly less than that expected from ionospheric dissipation. They concluded a waveguide/cavity mode was probably continuing to supply energy to the FLR. A rare sighting was reported by *Mann et al.* [1998], who observed an antisunward propagating waveguide mode wave packet. The amplitude of the wave packet was about 3 nT on a background field of 70 nT, indicating that nonlinear effects were not dominant. From these studies it is clear that the waveguide modes maintain a small amplitude, making them difficult to observe. They do not become nonlinear and result in a turbulent disrupted magnetospheric structure over the range of L shells through which they propagate.

2. Absolute and Convective Instabilities

The above situation for KH waveguide/cavity/body modes seems paradoxical: We have an unstable equilibrium, but disturbances in the body of the magnetosphere never grow to large amplitudes. This conundrum is based on normal mode concepts, where linear modes proportional to $\exp i(k_y y - \omega t)$ are sought (y is the azimuthal coordinate). These modes must satisfy a dispersion relation

$$D(\omega, k_y) = 0. \tag{1}$$

For real k_y the frequency $\omega = \omega_r + i\omega_i$ may be complex ($\omega_i < 0$, decaying in time; $\omega_i = 0$, oscillatory in time; or $\omega_i > 0$ unstable and growing in time.)

However, the time-dependent behavior of a dispersive system is not described accurately by a normal mode. The only way to determine the true solution is via an initial value problem where the state of a localized disturbance is defined at $t = 0$, $f(y, t = 0)$, and the subsequent evolution is solved for. Two different asymptotic behaviors of such a disturbance, or "wave packet," are shown in Figure 1. In Figure 1a the pulse $f(y, t = 0)$ grows in time and also broadens in space so that ultimately every point is disturbed. This situation is refered to as an "absolute" instability: At any fixed y (e.g., y_1), $f(y_1, t) \to \infty$ as $t \to \infty$.

The pulse in Figure 1a did not propagate very quickly in y. In Figure 1b the pulse propagates in y more rapidly than it broadens and grows. The result is that the wave packet, whose amplitude grows exponentially in time, is swept away sufficiently rapidly that at fixed $y = y_1$, $f(y_1, t) \to 0$ as $t \to \infty$. Thus the equilibrium is left unperturbed in a fixed region of space, despite supporting an unstable perturbation in that region! This situation is referred to as a "convective" instability.

Whether an instability is absolute or convective is frame-dependent. Consider how the wave packet in Figure 1a would appear to an observer at the origin of the y' coordinate system, which moves with speed V_f with respect to the y frame:

$$y' = y - V_f t \tag{2}$$

At fixed y' (e.g., $y' = 0$) the disturbance vanishes as $t \to \infty$, because the observer moves away from the wave packet faster than the wave packet broadens. In the y' frame we have a convective instability in Figure 1a. In Figure 1b an observer at $y' = 0$ tends to keep up with the propagating wave packet and sees $f(y' = 0, t) \to \infty$ as $t \to \infty$, so the instability is absolute in the y' frame. Transforming frame by V_f obviously Doppler-shifts the frequency, but, more importantly, is able to change the nature of the instability completely.

It was first noted by *Twiss* [1951, 1952], and subsequently by *Landau and Lifshitz* [1953] and *Sturrock* [1958], that the two classes of instabilities described

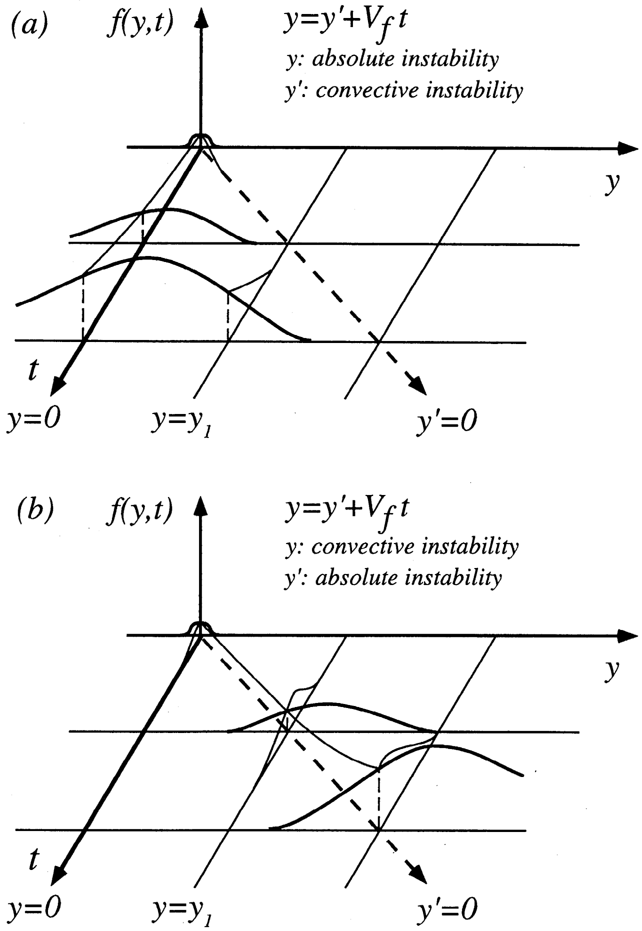

Figure 1. Schematic of the development of a localized unstable disturbance $f(y,t)$ in space and time,showing (a) the $f(y,t) \to \infty$ at any fixed y as $t \to \infty$ and (b) the wave packet moving sufficiently quickly that $f(y,t) \to 0$ at any fixed y as $t \to \infty$. The instability is absolute in Figure 1a and convective in Figure 1b for the y frame. The result is frame-dependent and may appear differently in the y' frame (adapted from Figure 3.2.1 of *Bers* [1983]).

above can be identified physically by the different behaviors. Recently, convectively unstable behavior has been discussed for the Kelvin-Helmholtz-driven magnetopause surface wave by *Manuel and Samson* [1993], *Miura* [1984], and *Wu* [1986]. These studies were numerical and did not exploit the analysis and classification of linear absolute and convective instabilities developed by previous researchers in other areas. Indeed, the magnetospheric community seems unaware of these studies and does not use the terms "convective" or "absolute" with the precise meaning with which *Briggs* [1964] coined them or the present paper imbues them. Nevertheless, in considering the stability of the magnetopause, the magnetospheric community has moved toward the notion of convective instabilities, and Manuel and Samson give a classic description of one without naming it explicitly.

The distinction between absolute and convective instabilities given above draws upon a consideration of the physical behavior of the initial disturbance for large t. There also exists a complementary and more mathematical distinction based on the behavior of the roots of the dispersion relation. This alternative analysis was originally developed for a laboratory plasma application by *Briggs* [1964]. It is surprising, perhaps, that magnetospheric plasma physicists are unaware of this work as we show that it has important consequences for magnetospheric Kelvin-Helmholtz-driven surface and waveguide modes. It seems worthwhile to summarize the main results of Briggs, and we do so in section 3.

3. Normal Modes and Wave Packets

A link between solutions of the dispersion relation (i.e., normal modes) and the development of a wave packet should not come as a surprise, since the subsequent evolution may be thought of as an initial value problem whose solution can be found in terms of the normal modes. The initial wave packet is expressed as a Fourier integral (along the real k_y axis) of the normal modes. The time dependence is found from a Laplace transform integrated in the complex frequency plane from $-\infty + i\sigma$ to $+\infty + i\sigma$ (the Bromwich contour), where σ is greater than the maximum growth rate of all the normal modes with real k_y. The solution of the initial value problem for localized disturbances represented by $f(y,t)$ is given by

$$f(y,t) = \int_{i\sigma-\infty}^{i\sigma+\infty} \int_{-\infty}^{\infty} \frac{T(\omega,k_y)}{D(\omega,k_y)} e^{i(k_y y-\omega t)} dk_y d\omega, \qquad (3)$$

where $T(\omega,k_y)$ depends on the initial perturbation.

The behavior of $f(y,t)$ as $t \to \infty$ for the wave packet can be different from that of a normal mode. The analysis of this problem has been developed extensively for other applications. We do not provide a detailed discussion here but refer the interested reader to, for example, the monograph by *Briggs* [1964] or the review by *Bers* [1983] and references therein. It turns out that double k roots of the dispersion relation determine the large t behavior of $f(y,t)$,

$$D(\omega,k_y) = \frac{\partial D(\omega,k_y)}{\partial k_y} = 0. \qquad (4)$$

Let one such root be located at complex ω and k_y, say, $\omega = \omega_o = \omega_{or} + i\gamma$ and $k = k_o$. In the vicinity of (ω_o, k_o) the dispersion relation has the form

$$(\omega - \omega_o)C^2 = (k - k_o)^2 \qquad (5)$$

and C is a complex constant. If the double root occurs for $\gamma < 0$, the point (ω_o, k_o) will not contribute to the instability, and the asymptotic behavior is $f(y,t) \to 0$ as $t \to \infty$. If the double root occurs for $\gamma > 0$ and letting $\omega_i \to +\sigma$ splits the double into two simple roots lying on opposite sides of the real k_y axis, then these roots are termed "pinching roots." Such a double root will contribute to the instability, and the asymptotic behavior of f is

$$f(y,t) \propto \exp[i(k_o y - \omega_{or} t)]\frac{e^{\gamma t}}{t^{1/2}}, \tag{6}$$

corresponding to unbounded growth at every point in space, as is observed in the y frame in Figure 1a. The dominant growth of $f(y,t)$ is associated with the suitable pinching double root (ω_o, k_o) having the largest $\gamma > 0$. The large t solution is given explicitly in (6). The situation in Figure 1b corresponds to a convective instability in the y frame, and all pinching double roots have $\gamma < 0$. The sign of γ provides a mathematical distinction between convective and absolute instabilities. In the case of a multiple k root of $D(\omega, k_y) = 0$ satisfying the above splitting (collision) condition, the asymptotic behavior is like that in (6) except that instead of $t^{1/2}$ in the denominator, t^s generally appears (with $s < 1/2$) [see *Brevdo*, 1988].

The physical discussion of section 2 showed how the convective and absolute nature of the instability could change between frames of reference. This may also be appreciated with the mathematical definition described above. The double roots in the y' frame will occur at (ω'_o, k'_o) which will be different from the double root in the rest frame (ω_o, k_o). Using the Doppler frame mappings

$$k_y = k'_y \qquad \omega = \omega' + k'_y V_f, \tag{7}$$

the associated double root in the y' frame must satisfy

$$D(\omega' + k'_y V_f, k'_y) = \frac{\partial D(\omega' + k'_y V_f, k'_y)}{\partial k'_y} = 0. \tag{8}$$

If $\gamma(V_f) = \mathrm{Im}\omega' > 0$, an expression similar to (6) gives the asymptotic behavior in the y' frame. For example, the absolute instability in the y frame in Figure 1a becomes a convective instability in the y' frame when the $\gamma(V_f)$ value of the double root shifts from the upper half of the ω' plane to the lower half.

4. Wave Packet Evolution

Figure 2 shows a simple model of the waveguide on the magnetospheric flanks [e.g., *Fujita et al.*, 1996; *Walker*, 1998; *Mann et al.*, 1999; *Mills et al.*, 1999]. The body of the magnetosphere is taken as a cold plasma with constant density ρ_1 permeated by a uniform magnetic field, while the magnetosheath is a field-free plasma of density ρ_2 flowing with constant speed.

Note that a discontinuity in flow speed would result in the KH surface mode having an infinite growth rate as $k_y \to \infty$ ($k_{yi} = 0$), which means that the stability problem is ill posed. In particular, this makes it impossible to place the Bromwich contour in equation (3) above all the ω roots of the dispersion relation for real k_y. We resolve this problem by assuming that the flow changes over a narrow boundary layer (similar to the low-latitute boundary layer) in the following discussion and computations. The calculation is performed in normalized units, and the flow changes smoothly [see *Walker*, 1981] over the boundary layer $1-\delta < x < 1+\delta$. The change in flow speed across the boundary layer is denoted by ΔV. For simplicity, other equilibrium quantities change from magnetospheric to magnetosheath values discontinuously at $x = 1-\delta$. This should provide a good representation of the dayside and flanks magnetosphere in the equatorial regions. An outgoing boundary condition is imposed on magnetosheath waves, and an inner reflecting boundary is placed in the magnetosphere to mimic the trapping of waveguide modes by refraction in a nonuniform magnetosphere (see *Mills et al.* [1999] for more details).

Wave packets in this model were treated by computing the roots of (8). This was done by solving these equations numerically with the outgoing boundary condition applied. Such a treatment necessarily involves calculating the normal modes, and we found two distinct types of mode present in our model: the surface mode and the body (or waveguide) mode. The amplitude of the surface mode is largest in the velocity boundary layer and decays exponentially away from it. On the other hand, the amplitude of the waveguide modes is most significant throughout the body of the magnetosphere where they are trapped.

4.1. KH Surface Mode Wave Packets

The locations of suitable double roots (ω'_o, k'_o) were found by solving equation (8) as a function of V_f. (The value of k_z is treated as a fixed parameter in (1), and for simplicity we take it to be zero from here on.) The resulting $\gamma(V_f)$ for the KH surface mode is shown in Figure 3. Values of V_f for which $\gamma > 0$ correspond to frames in which the instability is absolute. The maximum of γ occurs when V_f is equal to the group velocity of the normal mode having the maximum growth rate ($\omega_{i\max}$) for real k_y. In this situation the observer moves to keep up with the wave packet ($V_f = V_1$) and so sees the maximum growth rate, $\gamma(V_f = V_1) = \omega_{i\max}$ [*Bers*, 1983]. If the observer moves slightly faster or slower than this group velocity, the observed growth rate is

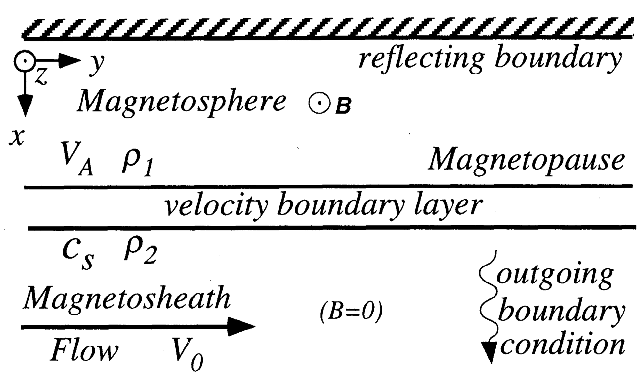

Figure 2. Model flank and magnetosheath equilibrium. The velocity shear layer at the magnetopause has a width of 2δ, across which the equilibrium flow changes by ΔV.

reduced. As the observer's V_f differs increasingly from the wave packet group velocity, γ is reduced to zero and is then negative (the speed of the observer now corresponding to that of a frame with a convective instability). The frame speed V_3 corresponds to the case where the wave packet just runs away from the observer, and V_2 corresponds to the situation where the observer just runs ahead of the wave packet; thus for large t no disturbance is seen in these frames. It is interesting that the discussion of the convectively unstable surface mode by *Wu* [1986] was based on a simulation in a frame moving with the wave packet's group velocity (equivalent to V_1). In this frame the instability, besides being absolute, has its maximum growth rate.

The magnetospheric rest frame corresponds to $V_f = 0$, so the KH surface mode wave packet is definitely convectively unstable ($\gamma < 0$) in the magnetospheric frame. This suggests that the equilibrium magnetopause may be disrupted by the unstable surface mode wave packet some distance downstream where it has grown to nonlinear amplitudes. To be sure of this, we need to calculate the length scale on which the wave packet grows as it convects in the magnetospheric rest frame [*Brevdo*, 1994].

Figure 3 tells us that in a frame moving with speed V_f, the amplitude of the wave packet grows in time as $\exp[\gamma(V_f)t]$ at fixed y'. Transforming back to the magnetospheric frame, this will appear as a spatial growth in $y = V_f t + y'$ given by $\exp[\gamma(V_f)y/V_f]$. Thus the e-folding length scale of spatial growth in the magnetospheric frame is

$$L_e(V_f) = V_f/\gamma(V_f). \qquad (9)$$

The $L_e(V_f)$ that will be observed is that corresponding to the fastest spatial growth, i.e., the minimum L_e. Minimizing (9) by taking d/dV_f and equating to 0 yields the condition

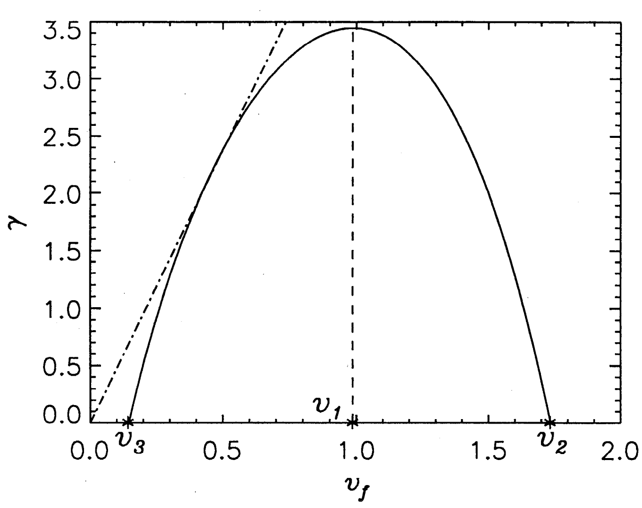

Figure 3. Asymptotic growth rate (γ) of the Kelvin-Helmholtz (KH) surface mode wave packet as a function of the observer's frame velocity (V_f). $V_f = 0$ corresponds to the magnetospheric rest frame ($\delta = 0.05$ and $\Delta V = 2.0$).

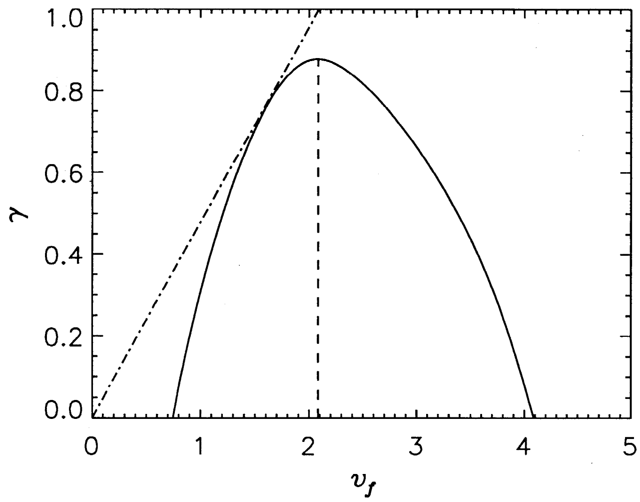

Figure 4. Asymptotic growth rate (γ) of the first-harmonic KH waveguide mode wave packet as a function of the observer's frame velocity (V_f). $V_f = 0$ corresponds to the magnetospheric rest frame ($\delta = 0.1$ and $\Delta V = 5.0$).

$$\frac{d\gamma}{dV_f} = \frac{\gamma}{V_f} \qquad (10)$$

and corresponds to the point where the tangent of $\gamma(V_f)$ goes through the origin. The slope of this line indicated in Figure 3 is the reciprocal of the minimum L_e.

4.2. KH Waveguide Mode Wave Packets

In our computations we found a sequence of unstable waveguide modes. The number of these modes increased with ΔV. Here we present the results for the case when only one waveguide mode, i.e., the first harmonic, is unstable. Figure 4 shows the asymptotic growth rate $\gamma(V_f)$ of a wave packet associated with the first-harmonic KH waveguide mode. The generic form and interpretation of this curve is the same as that of the KH surface mode (Figure 3). The KH waveguide mode wave packet is convectively unstable in the magnetospheric rest frame ($\gamma(V_f = 0) < 0$), and so we need to calculate the minimum L_e for this mode to decide if it becomes nonlinear on the magnetospheric flanks. The reciprocal of the gradient of the straight line indicated in Figure 4 gives the minimum L_e.

5. Discussion

The velocities in Figures 3 and 4 are normalised to the sheath sound speed (typically 100 km s^{-1}). Lengths are normalized by the penetration depth of waveguide modes into the magnetosphere (about 10 R_E), and time is normalized by the ratio of these quantities. The ratio of magnetospheric to magnetosheath plasma density is taken as 0.192 in the remainder of this paper.

5.1. KH Surface Mode Wave Packets

The KH surface mode is unstable for any change of flow speed across the boundary layer ΔV in our equilibrium model, and so it will start to grow near the subsolar point. We took $\delta = 0.05$ and $\Delta V = 2$ in constructing Figure 3, which represents a boundary layer width of 0.5 R_E and a sheath flow speed of 200 km s^{-1}. For these values the gradient of the line is 4.75, corresponding to a minimum L_e of 0.21, which in dimensional units is 2 R_E. Thus the KH surface mode wave packet will grow by at least 5 or 6 orders of magnitude as it propagates around the flanks traveling a path length of about 30 R_E. We would certainly expect this to become turbulent and experience a nonlinear saturation in the boundary layer before leaving the flank.

5.2. KH Waveguide Mode Wave Packets

The waveguide modes become KH unstable only if the sheath flow speed exceeds some threshold. Observations [*Engebretson et al.*, 1998] and theory [*Mann et al.*, 1999; *Mills et al.*, 1999] indicate that a flow speed of 500 km s^{-1} is sufficient for instability. This will not occur at the subsolar point but may well be satisfied on the flanks where the boundary layer may be 1.5 or 2 R_E wide. Hence suitable parameters for studying KH waveguide modes are $\Delta V = 5.0$ and $\delta = 0.1$, which were used to produce Figure 4. The gradient of the line in Figure 4 is 0.48, corresponding to a minimum growth length of about 2, or 20 R_E in dimensional units.

The length of magnetopause (in the equatorial plane on the dawn flank) from noon through dawn and to the beginning of the tail (say 0400 Magnetic Local Time (MLT)) is roughly 30 R_E. Beyond this point we would need to modify the equilibrium field and flow so that they have an orientation appropriate to the tail, rather than that used on the flank. (It is to be expected that the tail configuration will be more stable than the flank.) This suggests that the KH waveguide mode wave packet has at most 30 R_E to grow over and possibly less, since the waveguide modes will be stable around noon where the sheath flow speed will be less than $c_s + V_A$. It seems likely that wave packets of unstable waveguide modes will have at most roughly one e-folding length before they enter the tail, and this would seem to answer our original question of why unstable waveguide modes which propagate in the body of the magnetosphere do not disrupt and destroy the magnetospheric equilibrium; they propagate into the tail before growing to sufficient amplitude.

5.3. Other Studies

Most previous work has focused on the magnetopause surface wave, which is quite different from the magnetospheric waveguide modes. *Walker* [1981] and *Miura and Pritchett* [1982] considered the linear KH surface modes of the magnetopause. *Miura* [1984] went on to try and interpret these results in terms of spatial growth in the magnetospheric rest frame based on the maximum temporal growth rate ($\omega_{i\max}$) for real k_y and the associated phase velocity. This gives a growth length of $V_p/\omega_{i\max}$ (compare equation (9)). As we pointed out in section 5.1, double roots of the dispersion relation must be used to calculate this length, in particular, the straight line indicated in Figure 3. Interestingly, using $\omega_{i\max}$ and the group velocity of that mode would correspond to a straight line passing through the origin and the peak of the γ versus V_f curve. (*Briggs* [1964] shows that this peak happens to coincide with a double root of the dispersion relation.) However, this does not correspond to the fastest spatial growth. Depending on the location of the $\gamma(V_f)$ curve, such a line may provide a reasonable estimate of L_e, and this has often been employed in the past (see *Brevdo* [1994] for a more detailed discussion).

The most important equilibrium scale length for the KH surface modes is the width of the velocity shear layer, and it is useful to evaluate the quantity L_e/δ to facilitate an easy comparison of different calculations. *Miura* [1984] estimates $L_e/\delta = 12$.

The simulation by *Manuel and Samson* [1993] fed seed perturbations in at one end of the domain and showed how the surface mode grew exponentially in space as it convected away. This is a classic feature of convective instabilities. Indeed, if the equilibrium had been absolutely unstable, the disturbance would have grown exponentially in time everywhere, resulting in an unbounded solution. Manuel and Samson modeled their linear phase as a normal mode with real frequency and complex k_y in Earth's frame, which actually corresponds to an example of the classic "signaling problem." It is interesting to note that when this solution is viewed from a moving frame, not only is the frequency shifted but so is the growth rate in equation (7). From Figure 5a of Manuel and Samson we find $L_e/\delta = 9$. These results are in good general agreement with our double root calculation, which gives $L_e/\delta = 4.2$ for the KH surface wave. Some spread in these results should be expected owing to differences in the equilibrium models, etc.

Wu [1986] performed numerical simulations of KH unstable surface mode wave packets in the frame moving with the wave packet. This frame moves tailward with respect to the magnetospheric rest frame. Although Wu employs the term "convective instability" and mentions a finite tailward velocity of the wave packet, he clearly does not use the term convective in the precise sense that we do. Wu notes that his results can be transformed easily into any frame (e.g., the magnetospheric rest frame), but he does not do so, and this is an essential step in determining the absolute/convective nature of the instability. Physically, the distinction between a convective instability and an absolute instability can be appreciated in terms of competition between the wave packet propagation speed and the rate of wave packet broadening in space. For example, in Figure 1a the broadening dominates over the small propagation speed in the y frame, and the instability is absolute. Whereas in Figure 1b the larger propagation speed dominates the broadening in the y frame, and the instability is convective. Hence, Wu's tailward wave packet velocity in the magnetospheric rest frame does not guarantee that the instability is convective in this frame, since the speed at which the wave packet broadens must be allowed for. Wu's principal concern was the effect of periodic boundary conditions, and for the majority of his paper, convective could be read as nonperiodic.

Although the studies by *Miura* [1984], *Wu* [1986], and *Manuel and Samson* [1993] have been unaware of the literature describing how to characterize absolute and convective instabilities, they have clearly moved toward a general picture of a convectively unstable KH surface mode. Indeed, the results of Manuel and Samson constitute unambiguous proof of the instability's convective nature (in the sense that *Briggs* [1964] and we use the term).

KH waveguide normal modes have only been discussed recently in terms of single roots of the dispersion relation [*Fujita et al.*, 1996; *Mann et al.*, 1999; *Mills et al.*, 1999] which demonstrates that these modes (which propagate throughout the body of the magnetosphere) can be unstable on the flanks. To our knowledge, there has been no treatment of the absolute or convective nature of the KH waveguide wave packets until the present paper.

5.4. Normal Modes and Amplifying Waves

Traditionally, magnetospheric surface and waveguide modes have been studied through single roots of the dispersion relation, i.e., normal modes. In this paper we have shown how the double roots of the dispersion relation are important for determining the nature of the instability. In certain cases the normal modes may yield much information and physical understanding. However, this is not guaranteed, and we must first consider the location of the double roots to determine the usefulness of a normal mode. This is illustrated particularly clearly in the classical "signaling" problem.

So far we have considered the free response of our equilibrium to an initial condition. Now suppose we include a driving term at some fixed position (e.g., the subsolar point, say, $y = 0$) that varies as $\exp -i\omega_d t$ for $t > 0$ and is zero for $t < 0$. This is an example of the signaling problem. One type of behavior represents the driver exciting a wave train that is swept away from the driving location and grows exponentially in the direction of propagation. These are sometimes referred to as amplifying waves. In this situation the asymptotic solution is a normal mode with real frequency (ω_d) and a complex k_y. The alternative behavior corresponds to every fixed point being perturbed by a disturbance that grows exponentially with time and whose frequency is not related to ω_d. The normal mode with frequency ω_d is not relevant to the latter case.

The two types of behavior are actually manifestations of systems that are convectively or absolutely unstable. This may be appreciated by considering the plasma response given in (3). The inclusion of the driver described above modifies this to

$$f(y,t) = \int_{i\sigma-\infty}^{i\sigma+\infty} \int_{-\infty}^{\infty} \frac{\tilde{T}(\omega, k_y)}{(\omega - \omega_d) D(\omega, k_y)} e^{i(k_y y - \omega t)} dk_y d\omega, \tag{11}$$

where $\tilde{T}(\omega, k_y)$ depends on the initial perturbation and the spatial form of the driver. The integrand has an additional pole at $\omega = \omega_d$.

Figure 5a shows the path of the Laplace integral (the Bromwich contour (B) which passes through the point $i\sigma$) in the complex ω plane. Each point on the Bromwich contour may be mapped via the dispersion relation (1) onto a set of points in the complex k_y plane (Figure 5b). A frequency on the Bromwich contour and its k_y images are denoted by crosses in Figure 5. To

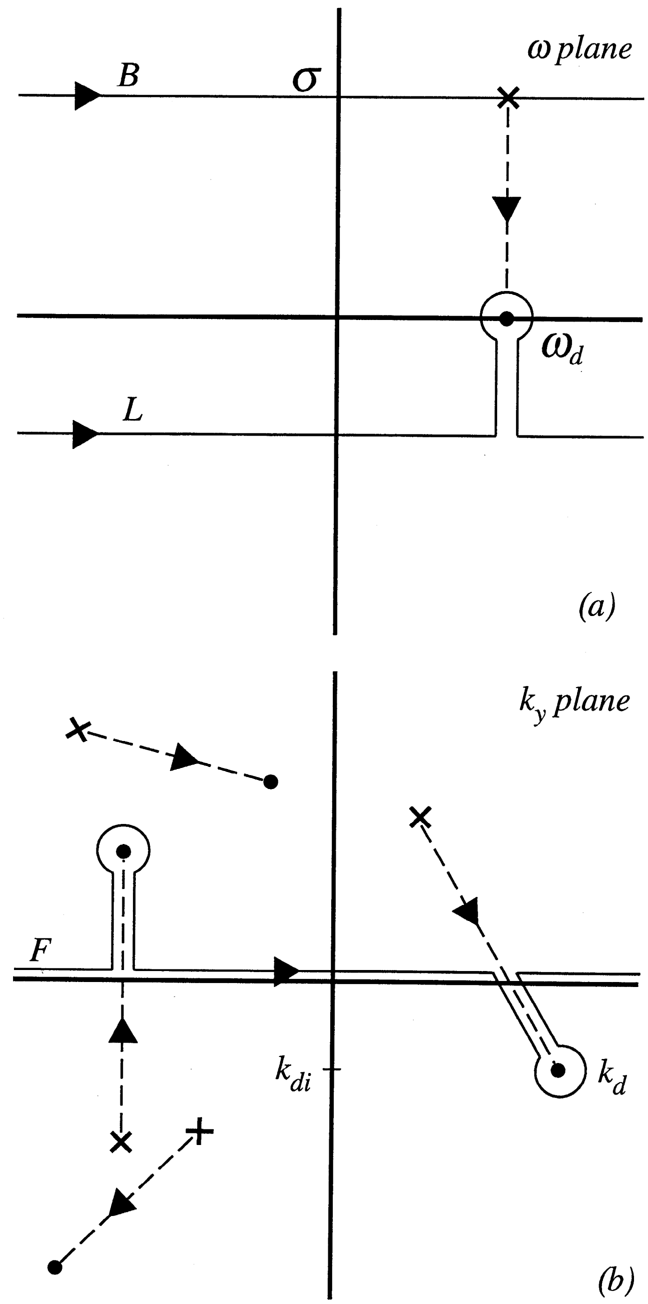

Figure 5. The path of (a) the Laplace integral in the complex ω plane, and (b) the Fourier integral in the complex k_y plane. The trajectory of a point in the ω plane and its images in the k_y plane are shown by dashed lines. A steady driving source introduces an additional pole at $\omega = \omega_d$ which may play an important role in determining the asymptotic behavior.

determine the asymptotic behavior in time, we try to lower the contour of the Laplace integral to the lower half of the ω plane. The trajectories of our selected frequency and its k_y images are shown by dashed lines in Figure 5. (Note that every point on the Bromwich contour must be lowered in turn.)

The Fourier integral in (11) is performed along the real k_y axis. As the Laplace contour is deformed and the roots in the k_y plane move, the Fourier contour (F in Figure 5b) must be distorted so that no root crosses it. This process can not be continued if a "pinching" double root is encountered for some $\omega_i > 0$. The asymptotic behavior is then given by (6) and corresponds to an absolute instability. In this situation the amplitude grows exponentially in time at every point. A steady state does not exist in this case, and the driving frequency ω_d is absent from the asymptotic behavior.

Different behavior is found if the pinching double roots all lie in the lower half of the ω plane. Now the Bromwich contour can be moved to below the real ω axis except for being distorted around the pole at ω_d (L in Figure 5a). Of the k_y trajectories in Figure 5b that cross the real axis, those which finish in the upper half correspond to growing waves in the region $y < 0$, and those in the lower half correspond to growing waves for $y > 0$. Suppose we are interested in the latter region. Of all the roots that cross to the lower half, we shall label the one with the most negative k_{yi} by k_d in Figure 5b. Writing $k_d = k_{dr} + ik_{di}$, this wavenumber will correspond to the shortest e-folding length in y of $1/k_{di}$, which will dominate the solution. The asymptotic solution in this case is proportional to

$$f(y,t) \propto \exp[i(k_{dr}y - \omega_d t)] \cdot e^{-k_{di}y}. \qquad (12)$$

This is identical to the single-root normal mode with frequency ω_d, and so in this case (where the equilibrium is convectively unstable), it is a useful way of interpreting the asymptotic time-dependent solution. Note that not all driving frequencies will produce this result. For some values of ω_d it may be the case that no k_y trajectories cross the real axis. In this case the normal mode does not grow in space, and a stable or dispersing wave train will be excited by the driver. Further discussion can be found in the work of *Briggs* [1964] and *Bers* [1983].

6. Conclusions

The surface and waveguide modes of the magnetopause and magnetosphere are known to be KH unstable. In this paper we have looked at the nature of the instability in the magnetospheric rest frame, i.e., whether it is absolute or convective. This was accomplished by applying, for the first time, the sound mathematical formalism developed by *Briggs* [1964] to magnetospheric ULF waves.

The KH surface mode is confined to the vicinity of the shear flow boundary layer/magnetopause. We find that this mode is convectively unstable, and our results are in general agreement with previous studies. The mode grows on a length scale of 0.5 to 4 R_E for boundary layer widths of 0.1 to 1 R_E, respectively. Thus slightly away from noon, we expect a rapid spatial growth and nonlinear evolution of the surface mode which should disrupt the local velocity shear in the boundary layer. *Manuel and Samson* [1993] suggest that this can account for the broadening of the low-latitude boundary layer away from noon.

The KH waveguide mode wave packets are not confined to the magnetopause but propagate deep into the magnetosphere. If these KH unstable modes grow to become nonlinear, we would expect them to disrupt the equilibrium magnetosphere not just at the magnetopause but throughout the domain over which they propagate. Waveguide modes are only KH unstable for high sheath flow speeds and so are most likely to grow on the flanks. On the flanks the KH surface mode will have become nonlinear, and the effect of nonlinear saturation here may be represented phenomenologically as a broad boundary layer. Our calculation shows that the KH waveguide modes are convectively unstable in the magnetospheric rest frame and grow over an e-folding length of about 20 R_E. Thus the waveguide modes will probably not have grown by more than a factor of e before they leave the flank region and enter the magnetotail, where a different equilibrium model is required.

It is now apparent why linear wave theory has been so successful at describing the generation, propagation, and coupling of such ULF waveguide modes to field line resonances: The waveguide modes do not generally grow to nonlinear amplitudes locally. This is also a partial explanation of why waveguide modes are so hard to see in spacecraft data: Their amplitudes are normally small.

In summary, we find that KH surface modes are convectively unstable and grow over a relatively short length, resulting in their nonlinear evolution and saturation. The apparent stability of the body of the magnetospheric flanks in the presence of unstable waveguide modes may be understood in terms of a convective instability also. Unstable waveguide disturbances have a relatively large growth length and so are convected away without experiencing significant growth, and leave an unperturbed equilibrium state ultimately.

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L. Brevdo, Ecole Supérieure de Mechanique de Marseille, Université de la Mediterranée, 13451 Marseille, Cedex 20, France.

K. J. Mills, M. S. Ruderman, and A. N. Wright, Department of Mathematical and Computational Sciences, University of St. Andrews, St. Andrews, Fife KY16 9SS, Scotland, U.K. (andy@dcs.stand.ac.uk)

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